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Abstract

We present a qualitative positional calculus which uses oriented points as basic entities. In contrast to often-used simple points, we consider objects that have an intrinsic direction. Having an intrinsic orientation is an important property of natural objects.

1 Introduction

Qualitative Reasoning about space abstracts from the physical world and enables computers to make predictions about spatial relations, even when a precise quantitative information is not available [2]. The two main trends in Qualitative Spatial Reasoning are topological reasoning about regions [2, 9] and positional reasoning about point configurations [3, 11]. Especially positional reasoning is important for robot navigation [8].

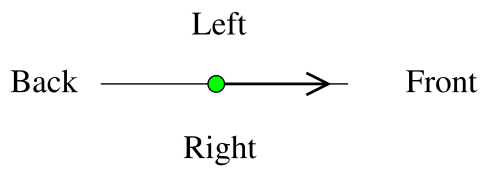

Figure 1: An oriented point and its qualitative spatial relative directions

In the aforementioned approaches about orientations objects and locations are represented as simple, featureless points. In contrast, our paper presents a positional calculus which uses more complex basic entities. It is based on objects which are represented as oriented points. It is closely related to a previously designed calulus which is based on straight line segments (dipoles) [7]. Conceptually our new calculus can be viewed as a transition from oriented line segements with concrete lenght to line segments with infinitely small length. In this conceptualisation the length of the objects is of no importance any longer. So only the direction of the objects is modelled. *O-points*, how we termed these oriented points, may be specified as pair of a point and a direction on the 2D-plane.

2 Reasoning with coarse o-point relations

In the coarsest representation a single o-point induces the sectors depicted in figure 1. "Front" and "Back" are linear sectors. "Left" and "Right" are half-planes. The position of the point itself is denoted as "Same". A qualitative spatial relative orientation relation between two o-points is represented by the sector in which the second o-point lies with respect to the frist one and by the sector in which the first one lies with respect to the second one.

For the general case of the two points having differnt positions we use the concatenated string of both sector names as relation symbol. Then the configuraton shown on figure 2 is expressed with the relation $A\ \mathrm{RightLeft}\ B$. If both points share the same position the relation symbol starts with the word "Same" and the second substring denotes the direction of the second o-point with respect to the first one as shown on figure 3.

The goal of identifying different relations is to obtain a set of jointly exhaustive and pairwise disjoint *atomic* relations such that between any two o-points exactly one relation holds. If these relations form a relation algebra it is possible to apply standard constraint-based reasoning mechanisms which were originally developed for temporal reasoning [1] and which have also proved valuable for spatial reasoning.

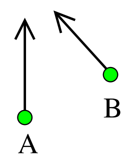

Figure 2: Qualitative spatial relation relation between two oriented points on different positions. The qualitative spatial reltions depicted here is $A\ \mathrm{RightLeft}\ B$

Altogether we obtain 20 different atomic relations (four times four general relations plus four with the o-points at the same position) . These relations are jointly exhaustive and pairwise disjoint. The relation SameFront is the identity relation. We use $\mathcal{OP}_1$ to refer to the set of 24 atomic relations, and $\mathcal{OPRA}_1$ to refer to the powerset of $\mathcal{OP}_1$ which contains all 2^{20} possible unions of the atomic relations.

For reasoning about the o-point relations we apply constraint-based reasoning techniques which were originally introduced for temporal reasoning [1] and which also proved valuable for spatial reasoning [9, 4]. In order to apply these techniques to a set of relations, these relations must form a relation algebra [5], i.e. they must be closed

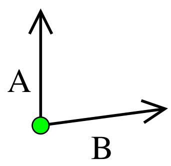

Figure 3: Qualitative spatial relation relation between two oriented points on the same position. The qualitative spatial reltions depicted here is A $\mathrm{SameRight}$ B

under composition ($\circ$), intersection ($\cap$), complement ($\bar{\cdot}$), and converse ($\smile$) and there must be an empty relation, a universal relation, and an identity relation. While the converse, the complement, and the intersection of relations can be computed from the set-theoretic definitions of the relations, the composition of relations must be computed based on the semantics of the relations. The compositions are usually computed only for the atomic relations which are then stored in a composition table. The composition of compound relations can be obtained as the union of the compositions of the corresponding atomic relations. The compositions of the atomic relations can be deduced directly from the geometric semantic of the relations. The composition table (as well as the converse table) for the atomic relations of the $\mathcal{OPRA}_1$ calculus can be obtained at [12].

O-point constraints are written as xRy where x, y are variables for o-points and R is a $\mathcal{OPRA}_1$ relation. Given a set Θ of o-point constraints, an important reasoning problem is deciding whether Θ is *consistent*, i.e., whether there is an assignment of all variables of Θ with dipoles such that all constraints are satisfied (a *solution*). We call this problem OPSAT. OPSAT is a Constraint Satisfaction Problem (CSP) [6] and can be solved using the standard methods developed for CSPs with infinite domains (see, e.g. [5]).

A partial method for determining inconsistency of a set of constraints Θ is the *path-consistency method* which enforces path-consistency on Θ [6]. A set of constraints is path-consistent if and only if for any two consistent variable instantiatons, there exists an instantiation of any third variable such that the three values taken together are consistent. It is necessary but not sufficient for the consistency of a set of constraints that path-consistency can be enforced. A naive way to enforce path-consistency is to strengthen relations by successively applying the following operation until a fixed point is reached:

$$\forall i, j, k : \quad R_{ij} \leftarrow R_{ij} \cap (R_{ik} \circ R_{kj})$$

where i, j, k are nodes and R_{ij} is the relation between i and j. The resulting set of constraints is equivalent to the original set, i.e. it has the same set of solutions. If the empty relation occurs while performing this operation Θ is inconsistent, otherwise the resulting set is path-consistent.

3 Finer grained o-point calculi

The design principle for $\mathcal{OPRA}_1$ can be generalized to calculi $\mathcal{OPRA}_m$. Then an angular resulution of $\frac{360}{2m}$ degree is used for the representation (a similar scheme for absolute direction instead of relative direction was recently designed by Renz and Mitra [10]).

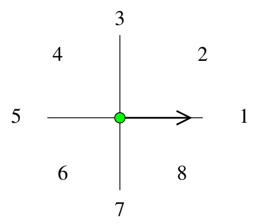

Figure 4: $\mathcal{OPRA}_2$ granularity

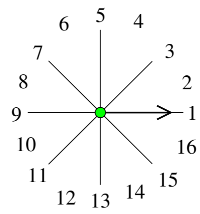

Figure 5: $\mathcal{OPRA}_4$ granularity

For formally specifying the o-point relations we use two-dimensional continuous space, in particular $\mathbb{R}^2$. Every o-point S on the plane is an ordered pair of a point $\mathbf{p}_S$ represented by its Cartesian coordinates x and y, with $x, y \in \mathbb{R}$ and and a direction ϕ_S.

$$S = (\mathbf{p}_S, \phi_S), \qquad \mathbf{p}_S = ((\mathbf{p}_S)_x, (\mathbf{p}_S)_y)$$

We distinguish the relative locations and orientations of the two o-points A and B expressed by a calculus $\mathcal{OPRA}_m$ according to the following scheme. We use the symbol φ_{AB} for $\tan^{-1} \frac{(\mathbf{p}_B)_y - (\mathbf{p}_A)_y}{(\mathbf{p}_B)_x - (\mathbf{p}_A)_x}$ ($\tan^{-1}$ has two arguments, the numerator, and the denominator, and maps to the interval $[0, 2\pi]$). If $\mathbf{p}_A \neq \mathbf{p}_B$ the relation $A \ {}_m\angle_j^i \ B$ represents the following set of configurations:

$$
\begin{aligned}
& \left(\left(\left(\frac{i}{2} \in \mathbb{N} \wedge i \geq 2 \right) \wedge \left(2\pi \frac{i-2}{4m} < \varphi_{AB} - \phi_A < 2\pi \frac{i}{4m} \right) \right) \right. \\
\vee \quad & \left. \left(\left(\frac{i+1}{2} \in \mathbb{N} \wedge i \geq 1 \right) \wedge \left(\varphi_{AB} - \phi_A = 2\pi \frac{i-1}{4m} \right) \right) \right) \\
\wedge \quad & \left(\left(\left(\frac{j}{2} \in \mathbb{N} \wedge j \geq 2 \right) \wedge \left(2\pi \frac{j-2}{4m} < \varphi_{AB} - \phi_B < 2\pi \frac{j}{4m} \right) \right) \right. \\
\vee \quad & \left. \left(\left(\frac{j+1}{2} \in \mathbb{N} \wedge j \geq 1 \right) \wedge \left(\varphi_{AB} - \phi_B = 2\pi \frac{j-1}{4m} \right) \right) \right)
\end{aligned}
$$

Using this notation a simple manipulation of the parameters yields the converse operation $({}_m\angle^i_j)^\smile = {}_m\angle^j_i$. If $\mathbf{p}_A = \mathbf{p}_B$ the relation $A\ {}_m\angle i\ B$ represents the following set of configurations:

$$
\begin{aligned}
& \left(\left(\frac{i+1}{2} \in \mathbb{N} \wedge i \geq 1 \right) \wedge \left(\phi_B - \phi_A = 2\pi \frac{i-1}{4m} \right) \right) \\
\vee \quad & \left(\left(\frac{i}{2} \in \mathbb{N} \wedge i \geq 2 \right) \wedge \left(2\pi \frac{i-2}{4m} < \phi_B - \phi_A < 2\pi \frac{i}{4m} \right) \right)
\end{aligned}
$$

Using this notation a simple manipulation of the parameters yields the converse operation $({}_m\angle i)^\smile = {}_m\angle(4m - i)$. The composition tables for the atomic relations of the $\mathcal{OPRA}_m$ calculi can be generated using a schemata which is based on the parameters m, i, j of the corresponding relations (analogous to the generating scheme for the converse operation). The schemata for the composition operation can be obtained at [12].

4 Conclusion

We presented a calculus for representing and reasoning about qualitative relative orientation information. Oriented points serve as the basic entities since they are the simplest spatial entities that have an intrinsic orientation. We identified systems of atomic relations on different granularity levels between o-points and identified a scheme for computing the calculi's operation tables based on their geometric semantics, which allows for applying constraint-based reasoning methods.

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