

SparQ: A Toolbox for Qualitative Spatial Representation and Reasoning

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Abstract. A multitude of calculi for qualitative spatial reasoning (QSR) has been proposed during the last two decades. The number of practical applications that make use of QSR techniques is, however, comparatively small. One reason for this may be seen in the difficulty for people from outside the field to incorporate the required reasoning techniques into their software. Sometimes, proposed calculi are only partially specified and implementations are rarely available. With the SparQ toolbox presented in this text, we seek to improve this situation by making common calculi and standard reasoning techniques accessible in a way that allows for easy integration into applications. We hope to turn this into a community effort and encourage researchers to incorporate their calculi into SparQ. This text provides an overview on SparQ and its utilization.

1 Introduction

Qualitative spatial reasoning (QSR) is an established field of research pursued by investigators from many disciplines including geography, philosophy, computer science, and AI [1]. The general goal is to model commonsense knowledge and reasoning about space as efficient representation and reasoning mechanisms that are still expressive enough to solve a given task.

Following the approach taken in Allen’s seminal paper on qualitative temporal reasoning [2], QSR is typically realized in form of calculi over sets of spatial relations (like ‘left-of’ or ‘north-of’). These are called *qualitative spatial calculi*. A multitude of spatial calculi has been proposed during the last two decades, focusing on different aspects of space (mereotopology, orientation, distance, etc.) and dealing with different kinds of objects (points, line segments, extended objects, etc.). Two main research directions in QSR are mereotopological reasoning about regions [3,4,5] and reasoning about positional information (distance and orientation) of point objects [6,7,8,9,10] or line segments [11,12,13].

Despite this huge variety of qualitative spatial calculi, the amount of applications employing qualitative spatial reasoning techniques is comparatively small.

We believe that important reasons for this are the following: Choosing the right calculus for a particular application is a challenging task, especially for people not familiar with QSR. Calculi are often only partially specified and usually

no implementation is made available—if the calculus is implemented at all and not only investigated theoretically. As a result, it is not possible to “quickly” evaluate how different calculi perform in practice. Even if an application developer has decided on a particular calculus, he has to invest serious efforts to include the calculus and required reasoning techniques into the application. For many calculi this is a time-consuming and error-prone process (e.g. involving writing down huge composition tables, which are often not even completely specified in the literature).

Overall, we are convinced that the QSR community should strive for making the fruits of its work available to the public in a homogeneous framework. We have thus started the development of a qualitative spatial reasoning toolbox called *SparQ*¹ that aims at supporting the most common tasks—qualification, computing with relations, constraint-based reasoning, etc. (cp. section 3)—for an extensible set of spatial calculi. A complementary approach aiming at the specification and investigation of the interrelations between calculi has been described in [14]. Here, the calculi are defined in the algebraic specification language CASL. In contrast, our focus is on providing an implementation of QSR techniques that is tailored towards the needs of application developers. In its current version, SparQ mainly concentrates on calculi from the area of reasoning about the orientation of point objects or line segments. However, specifying and adding new calculi is in most cases very simple. We hope to turn this into a community effort, encouraging researchers from other groups to incorporate their own calculi.

In this text, we describe SparQ and its utilization. The current version of SparQ and further documentation will be made available at the SparQ homepage². The next section briefly recapitulates the relevant terms concerning QSR and spatial calculi as needed for the remainder of the text. In section 3, we describe the services provided by SparQ. Section 4 explains how new calculi can be incorporated into SparQ and section 5 describes how SparQ can be integrated into own applications.

2 Reasoning with qualitative spatial relations

A qualitative spatial calculus defines operations on a finite set \mathcal{R} of spatial relations. The spatial relations are defined over a particular set of spatial objects, the domain D (e.g. points in the plane, oriented line segments, etc.). While a *binary calculus* deals with binary relations $R \subseteq D \times D$, a *ternary calculus* operates with ternary relations $R \subseteq D \times D \times D$.

A spatial calculus establishes a set of typically jointly exhaustive and pairwise disjoint (JEPD) base relations \mathcal{BR} . The set \mathcal{R} of all relations considered by the calculus contains at least the base relations, the empty relation \emptyset , the universal relation U , and the identity relation Id ; the set \mathcal{R} should be closed under the

¹ **S**patial **R**easoning done **Q**ualitatively

² <http://www.sfbtr8.uni-bremen.de/project/r3/sparq/>

operations defined in the following. Typically, the powerset of the base relations $2^{\mathcal{BR}}$ is chosen for \mathcal{R} .³

As the relations are subsets of tuples from the same Cartesian product, the set operations union, intersection, and complement can be directly applied:

Union: $R \cup S = \{ x \mid x \in R \vee x \in S \}$

Intersection: $R \cap S = \{ x \mid x \in R \wedge x \in S \}$

Complement: $\bar{R} = U \setminus R = \{ x \mid x \in U \wedge x \notin R \}$

where R and S are both n -ary relations on D . The other operations depend on the arity of the calculus.

2.1 Operations for binary calculi

For binary calculi, two other important operations are required:

Converse: $R^\sim = \{ (y, x) \mid (x, y) \in R \}$

(Strong) composition: $R \circ S = \{ (x, z) \mid \exists y \in D : ((x, y) \in R \wedge (y, z) \in S) \}$

For some calculi no finite set of relations exists that includes the base relations and is closed under composition as defined above. In this case, a weak composition is defined instead that takes the union of all base relations that have a non-empty intersection with the result of the strong composition:

Weak composition: $R \circ_{weak} S = \{ d \mid T \in \mathcal{BR} \wedge d \in T \wedge T \cap (R \circ S) \neq \emptyset \}$

2.2 Operations for ternary calculi

While there is only one possibility to permute the two objects of a binary relation which leads to the converse operation, there exist 5 such permutations for the three objects of a ternary relation. This results in the following operations⁴ [15]:

Inverse: $INV(R) = \{ (y, x, z) \mid (x, y, z) \in R \}$

Short cut: $SC(R) = \{ (x, z, y) \mid (x, y, z) \in R \}$

Inverse short cut: $SCI(R) = \{ (z, x, y) \mid (x, y, z) \in R \}$

Homing: $HM(R) = \{ (y, z, x) \mid (x, y, z) \in R \}$

Inverse homing: $HMI(R) = \{ (z, y, x) \mid (x, y, z) \in R \}$

Composition for ternary calculi is defined accordingly to the binary case:

(Strong) comp.: $R \circ S = \{ (w, x, z) \mid \exists y \in D : ((w, x, y) \in R \wedge (x, y, z) \in S) \}$

Other ways of composing two ternary relations can be expressed as a combination of the unary permutation operations and the composition [16] and thus do not have to be defined separately. The definition of weak composition is identical to the binary case.

³ Unions of relations correspond to disjunction of relational constraints and thus we will often simply speak of disjunctions of relations as well and write them as sets $\{R_1, \dots, R_n\}$.

⁴ It is not needed to specify all these operations as some can be expressed by others.

2.3 Constraint reasoning with spatial calculi

Spatial calculi are often used to formulate constraints about the spatial configurations of a set of objects from the domain of the calculus as a constraint satisfaction problem (CSP): Such a spatial constraint satisfaction problem then consists of a set of variables X_1, \dots, X_n (one for each spatial object) and a set of constraints C_1, \dots, C_m (relations from the calculus). Each variable X_i can take values from the domain of the utilized calculus. CSPs are often visualized as constraint networks which are graphs with nodes corresponding to the variables and arcs corresponding to constraints. A CSP is consistent, if an assignment for all variables to values of the domain can be found, that satisfies all the constraints. Spatial CSPs usually have infinite domains and thus backtracking over the domains can not be used to determine global consistency.

Besides global consistency, weaker forms of consistency called *local consistencies* are of interest in QSR. On the one hand, they can be employed as a forward checking technique reducing the CSP to a smaller equivalent one (one that has the same set of solutions). Furthermore, in some cases they can be proven to be not only necessary but also sufficient for global consistency for the set \mathcal{R} of relations of a given calculus. If this is only the case for a certain subset \mathcal{S} of \mathcal{R} and this subset exhaustively splits \mathcal{R} (which means that every relation from \mathcal{R} can be expressed as a disjunction of relations from \mathcal{S}), this at least allows to formulate a backtracking algorithm to determine global consistency by recursively splitting the constraints and using the local consistency as a decision procedure for the resulting CSPs with constraints from \mathcal{S} [17].

One important form of local consistency is *path-consistency* which (in binary CSPs) means that for every triple of variables each consistent evaluation of the first two variables can be extended to the third variable in such a way that all constraints are satisfied. Path-consistency can be enforced syntactically based on the composition operation (for instance with the algorithm by van Beek [18]) in $O(n^3)$ time where n is the number of variables. However, this syntactic procedure does not necessarily yield the correct result with respect to path-consistency as defined above. Whether this is the case or not needs to be investigated for each individual calculus.

2.4 Supported calculi

The calculi currently included in SparQ are the FlipFlop Calculus (FFC) [8] with the \mathcal{LR} refinement described in [19], the Single Cross Calculus (SCC) and Double Cross Calculus (DCC) [6], the coarse-grained variant of the Dipole Relation Algebra ($\mathcal{DR}\mathcal{A}_c$) [11,12], the Oriented Point Relation Algebra \mathcal{OPRA}_m [9], as well as RCC-5 and RCC-8⁵ [3]. An overview is given in Table 1 where the calculi are classified according to their arity (binary, ternary), their domain (points, oriented points, line segments, regions), and the aspect of space modeled (orientation, distance, mereotopology). As can be seen, mainly calculi for

⁵ Currently only the relational specification is available for RCC, but no qualifier.

Calculus	arity		domain				aspect of space		
	binary	ternary	point	or. point	line seg.	region	orient.	dist.	mereot.
FFC/ \mathcal{LR}		✓	✓				✓		
SCC		✓	✓				✓		
DCC		✓	✓				✓		
$\mathcal{DR}\mathcal{A}_c$	✓				✓		✓		
$\mathcal{OPR}\mathcal{A}_m$	✓			✓			✓		
RCC-5/8	✓					✓			✓

Table 1. The calculi currently included in SparQ

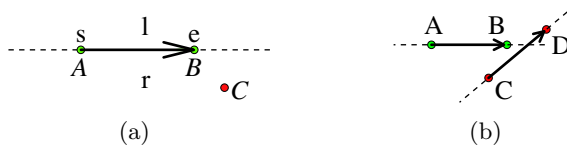


Fig. 1. Illustration of $\mathcal{DR}\mathcal{A}_c$: (a) The FlipFlop relations used to define Dipole relations. (b) A dipole configuration: $\mathbf{d}_{AB} rlll \mathbf{d}_{CD}$ in $\mathcal{DR}\mathcal{A}_c$.

reasoning about orientation have been incorporated so far, but calculi dealing with other kinds of base objects or dealing with other aspects of space can be integrated just as easily. We briefly describe $\mathcal{DR}\mathcal{A}_c$ as it will be used in the examples in the remainder of this text.

$\mathcal{DR}\mathcal{A}_c$ A dipole is an oriented line segment as e.g. determined by a start and an end point. We will write \mathbf{d}_{AB} for a dipole defined by start point A and end point B . The idea of using dipoles was first introduced by Schlieder [11] and extended in [12]. The coarse-grained dipole calculus variant ($\mathcal{DR}\mathcal{A}_c$) describes the orientation relation between two dipoles \mathbf{d}_{AB} and \mathbf{d}_{CD} with the preliminary of A, B, C , and D being in general position, i.e., no three disjoint points are collinear. Each base relation is a 4-tuple (r_1, r_2, r_3, r_4) of FlipFlop relations relating one point from one of the dipoles with the other dipole. r_1 describes the relation of C with respect to the dipole \mathbf{d}_{AB} , r_2 of D with respect to \mathbf{d}_{AB} , r_3 of A with respect to \mathbf{d}_{CD} , and r_4 of B with respect to \mathbf{d}_{CD} . The distinguished FlipFlop relations are left, right, start, and end (see Figure 1 (a)). Dipole relations are usually written without commas and parentheses, e.g. $rlll$. Thus, the example in Figure 1 (b) shows the relation $\mathbf{d}_{AB} rlll \mathbf{d}_{CD}$.

3 SparQ

SparQ consists of a set of modules that provide different services required for QSR that will be explained below. These modules are glued together by a central script that can either be used directly from the console or included into own applications via TCP/IP streams in a server/client fashion (see section 5).

The general syntax for using the SparQ main script is as follows:

```
$ ./sparq <module> <calculus identifier> <module specific parameters>
```

Example:

```
$ ./sparq compute-relation dra-24 complement "(lrl1 llrr)"
```

where ‘compute-relation’ is the name of the module to be utilized, in this case the module for conducting operations on relations, ‘dra-24’ is the SparQ identifier for the dipole calculus $\mathcal{DR}\mathcal{A}_c$, and the rest are module specific parameters, here the name of the operation that should be conducted (*complement*) and a string parameter representing the disjunction of the two dipole base relations *lrl1* and *llrr*⁶. The example call thus computes the complement of the disjunction of these two relations. SparQ provides the following modules:

qualify transforms a quantitative geometric description of a spatial configuration into a qualitative description based on one of the supported calculi

compute-relation applies the operations defined in the calculi specifications (intersection, union, complement, converse, composition, etc.) to a set of spatial relations

constraint-reasoning performs computations on constraint networks

Further modules are planned as future extensions. This comprises a quantification module for turning qualitative scene descriptions back into quantitative geometric descriptions and a module for neighborhood-based spatial reasoning. In the following section we will take a closer look at the three existent modules.

3.1 Qualification and scene descriptions

The purpose of the qualify module is to turn a quantitative geometric scene description into a qualitative scene description composed of base relations from a particular calculus. The calculus is specified via the calculus identifier that is passed with the call to SparQ. Qualification is required for applications in which we want to perform qualitative computations over objects whose geometric parameters are known.

The qualify module reads a quantitative scene description and generates a qualitative description. A quantitative scene description is a space-separated list of base object descriptions enclosed in parentheses. Each base object description is a tuple consisting of an object identifier and object parameters that depend on the type of the object. For instance, let us say we are working with dipoles which are oriented line segments. The object description of a dipole is of the form ‘(name x_s y_s x_e y_e)’, where name is the identifier of this particular dipole object and the rest are the coordinates of start and end point of the dipole. Let us consider the example in Figure 2 which shows three dipoles A, B, and C. The quantitative scene description for this situation would be:

⁶ Disjunctions of base relations are always represented as a space-separated list of the base relations enclosed in parentheses in SparQ.

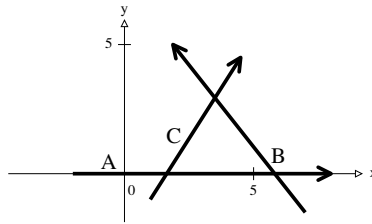


Fig. 2. An example configuration of three dipoles.

```
( ( A -2 0 8 0 ) ( B 7 -2 2 5 ) ( C 1 -1 4.5 4.5 ) )
```

The qualify module has one module specific parameter:

mode This parameter controls which relations are included into the qualitative scene description: If the parameter is 'all', the relation between every object and every other object will be included. If it is 'first2all' only the relations between the first and all other objects are computed.

The resulting qualitative scene description is a space-separated list of relation tuples enclosed in parentheses. A relation tuple consists of an object identifier followed by a relation name and another object identifier, meaning that the first object stands in this particular relation with the second object. The command to produce the qualitative scene description followed by the result is⁷:

```
$ ./sparq qualify dra-24 all
$ ( ( A -2 0 8 0 ) ( B 7 -2 2 5 ) ( C 1 -1 4.5 4.5 ) )
> ( ( A rllr B ) ( A rllr C ) ( B llrl C ) )
```

3.2 Computing with relations

The compute-relation module allows to compute with the operations defined in the calculus specification. The module specific parameters are the operation that should be conducted and one or more input relations depending on the arity of the operation. Let us say we want to compute the converse of the *llrl* dipole relation. The corresponding call to SparQ and the result are:

```
$ ./sparq compute-relation dra-24 converse llrl
> (rl11)
```

The result is always a list of relations as operations often yield a disjunction of base relations. The composition of two relations requires one more relation as parameter because it is a binary operation, e.g.:

```
$ ./sparq compute-relation dra-24 composition llrr rllr
> (lrrr llrr rlrr slsr lllr rllr rlll ells llll lr11)
```

⁷ In all the examples, input lines start with '\$'. Output of SparQ is marked with '>'.

Here the result is a disjunction of 10 base relations. It is also possible to have disjunctions of base relations as input parameters. For instance, the following call computes the intersection of two disjunctions:

```
$ ./sparq compute-relation dra-24 intersection "(rrrr rrll rllr)"
"(llll rrll)"
> (rrll)
```

3.3 Constraint reasoning

The constraint-reasoning module reads a description of a constraint network—which is a qualitative scene description that may include disjunctions and may be inconsistent and/or underspecified—and performs a particular kind of consistency check⁸. Which type of consistency check is executed depends on the first module specific parameter:

action The two actions currently provided are ‘path-consistency’ and ‘scenario-consistency’ and determine which kind of consistency check is performed.

The action ‘path-consistency’ causes the module to enforce path-consistency on the constraint network using van Beek’s algorithm [18] or detect the inconsistency of the network in the process. We could for instance check if the scene description generated by the qualify module in section 3.1 is path-consistent which of course it is. To make it slightly more interesting we add the base relation *ells* to the constraint between *A* and *C* resulting in a constraint network that is not path-consistent:

```
$ ./sparq constraint-reasoning dra-24 path-consistency
$ ( (A rllr B) (A (ells rllr) C) (B lrrl C) )
> Modified network.
> ( (B (lrrl) C) (A (rllr) C) (A (rllr) B) )
```

The result is a path-consistent constraint network in which *ells* has been removed. The output ‘Modified network’ indicates that the original network was not path-consistent and had to be changed. Otherwise, the result would have started with ‘Unmodified network’. In the next example we remove the relation *rllr* from the disjunction between *A* and *C*. This results in a constraint network that cannot be made path-consistent which implies that it is not globally consistent.

```
$ ./sparq constraint-reasoning dra-24 path-consistency
$ ( (A rllr B) (A ells C) (B lrrl C) )
> Not consistent.
> ( (B (lrrl) C) (A () C) (A (rllr) B) )
```

⁸ The constraint-reasoning module also provides some basic actions to manipulate constraint networks that are not further explained in this text. One example is the ‘merge’ operation that is used in the example in section 5.

SparQ correctly determines that the network is inconsistent and returns the constraint network in the state in which the inconsistency showed up (indicated by the empty relation $()$ between A and C).

If ‘scenario-consistency’ is provided as argument, the constraint-reasoning module checks if a path-consistent scenario exists for the given network. It uses a backtracking algorithm to generate all possible scenarios and checks them for path-consistency as described above. A second module specific parameter determines what is returned as the result of the search:

return This parameter determines what is returned in case of a constraint network for which path-consistent scenarios can be found. It can take the values ‘first’ which returns the first path-consistent scenario, ‘all’ which returns all path-consistent scenarios, and ‘interactive’ which returns one solution and allows to ask for the next solution until all solutions have been iterated.

Path-consistency is also used as a forward-checking method during the search to make it more efficient. For certain calculi, the existence of a path-consistent scenario implies global consistency. However, this again has to be investigated for each calculus. As a future extension it is planned to allow to specify splitting subsets of a calculus for which path-consistency implies global consistency and provide a variant of the backtracking algorithm that decides global consistency by searching for path-consistent instantiations that only contain relations from the splitting subset. In the following example, we use ‘first’ as additional parameter so that only the first solution found is returned:

```
$ ./sparq constraint-reasoning dra-24 scenario-consistency first
$ ( (A rele C) (A ells B) (C errs B) (D srs1 C) (A rser D) (D rrr1 B) )
> ( (B (rlrr) D) (C (slsr) D) (C (errs) B) (A (rser) D) (A (ells) B)
  (A (rele) C) )
```

In case of an inconsistent constraint network, SparQ returns ‘Not consistent.’

4 Specifying calculi in SparQ

For most calculi it should be rather easy to include them into SparQ. The main thing is to provide the calculus specification. Listing 1.1 shows an extract of the definition of a simple exemplary calculus for reasoning about distances between three point objects distinguishing the three relations ‘closer’, ‘farther’, and ‘same’. The specification is done in Lisp-like syntax.

The arity of the calculus, the base relations, the identity relation and the different operations have to be specified, using lists enclosed in parentheses (e.g. when an operation returns a disjunction of base relations). In this example, the inverse operation applied to ‘same’ yields ‘same’ and composing ‘closer’ and ‘same’ results in the universal relation written as the disjunction of all base relations. Some operations like homing and short cut operations are left out in the example (cmp. section 2.2).

```

(def-calculus "Relative distance calculus (reldistcalculus)"
  :arity :ternary
  :base-relations (same closer farther)
  :identity-relation same
5  :inverse-operation ((same same)
                      (closer closer)
                      (farther farther))
  :composition-operation ((same same (same closer farther))
                          (same closer (same closer farther))
10                          (same farther (same closer farther))
                          (closer same (same closer farther))
                          (closer closer (same closer farther))
                          (closer farther (same closer farther))
                          [...])

```

Listing 1.1. Specification of a simple ternary calculus (excerpt).

In addition to the calculus specification, it is necessary to provide the implementation of a qualifier function which for an n -ary calculus takes n geometric objects of the corresponding base type as input and returns the relation holding between these objects. The qualifier function encapsulates the methods for computing the qualitative relations from quantitative geometric descriptions. If it is not provided, the qualify module will not work for this calculus.

For some calculi, it is not possible to provide operations in form of simple tables as in the example. For instance, \mathcal{OPRA}_m has an additional parameter that specifies the granularity and influences the number of base relations. Thus, the operations can only be provided in procedural form, meaning the result of the operations are computed from the input relations when they are required. For these cases, SparQ allows to provide the operations as implemented functions and uses a caching mechanism to store often required results.

5 Integrating SparQ into own applications

SparQ can also run in server mode which makes it easy to integrate it into own applications. We have chosen a client/server approach as it allows for straightforward integration independent of the programming language used for implementing the application.

When run in server mode, SparQ takes TCP/IP connections and interacts with the client via simple plain-text line-based communication. This means the client sends commands which consist of everything following the ‘./sparq’ in the examples in this text, and can then read the results from the TCP/IP stream.

An example is given in Listing 1.2: A Python script opens a connection to the SparQ-server and performs some simple computations (qualification, adding another relation, checking for path-consistency). It produces the following output:

```

> ( (A rr11 B) (A rr11 C) )
> ( (A rr11 B) (A rr11 C) (B eses C) )
> Not consistent.
> ( (B (eses) C) (A () C) (A (rr11) B) )

```

```

# connect to sparq server on localhost, port 4443
sock = socket.socket(socket.AF_INET, socket.SOCK_STREAM)
sock.connect(('localhost', 4443))
sockfile = sock.makefile('r')
5 # qualify a geometrical scenario with DRA-24
sock.send('qualify dra-24 first2all ')
sock.send('((A 4 6 9 0.5) (B -5 5 0 2) (C -4 5 6 0))')
scene = readline() # read the answer
print scene
10 # add an additional relation (B eses C)
sock.send("constraint-reasoning dra-24 merge")
sock.send(scene + '(B eses C)')
scene2 = readline() # read the answer
print scene2
15 # check the new scenario for consistency
sock.send('constraint-reasoning dra-24 path-consistency')
sock.send(scene2)
print readline() # print the answer
print readline() # print the resulting constraint network

```

Listing 1.2. Integrating SparQ into own applications: an example in Python

6 Conclusions & outlook

The SparQ toolbox presented in this text is a first step towards making QSR techniques and spatial calculi accessible to a broader range of application developers. We hope that this initiative will catch interest in the QSR community and encourage other researchers to incorporate their calculi into SparQ.

Besides including more calculi, extensions currently planned for SparQ are a module for neighborhood-based reasoning techniques [20,13] (e.g. for relaxing inconsistent constraint networks based on conceptual neighborhoods and for qualitative planning) and a module that allows quantification (turning a consistent qualitative scene description back into a geometric representation). This requires the mediation between the algebraic and geometric aspects of a spatial calculus together with the utilization of prototypes. Moreover, we want to include geometric reasoning techniques based on Gröbner bases as a service for calculus developers as these can for instance be helpful to derive composition tables [12]. The optimization of the algorithms included in SparQ is another issue that we want to grant more attention to in the future. Finally, we intend to incorporate interfaces that allow to exchange calculus specifications with other QSR frameworks (e.g. [14]).

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