A Causal Perspective to Qualitative Spatial Reasoning in the Situation Calculus

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Abstract. We propose the utilisation of a general formalism to reason about action & change for reasoning about the dynamic purpose-directed aspects of spatial change. Such an approach is necessary toward the general integration of qualitative spatial reasoning with reasoning about the teleological aspects of spatial change. With this as the overall context, the main contribution of this paper is to illustrate first ideas relevant to providing a causal perspective to qualitative spatial reasoning using the situation calculus. With minimal notions about space & spatial dynamics, we perform a naive characterisation of objects based on their physical properties and investigate the key representational aspects of a topological theory of space, namely the region connection calculus, in the situation calculus. Further, ontological distinctions are made between various occurrents, i.e., actions and internal & external events, and a domain level characterisation of spatial occurrents in terms of their spatial pre-conditions & effects is performed so as to provide a causal perspective to spatial reasoning.

1 Introduction

Qualitative spatial reasoning (QSR) is an abstraction from precision oriented quantitative reasoning about the physical world [1]. Most research in qualitative spatial reasoning has focused on particular aspects of space such as topology [12], orientation [5], distance [7] etc and their integration thereof. However, relatively little work has explicitly addressed the need to account for the dynamic teleological or purpose-directed aspects of spatial change within a unified setup. Whereas qualitative spatial reasoning is concerned with the manner in which a set of spatial relationships evolve during a certain time interval, reasoning about the teleological aspects of a system involves reasoning about action and encompasses the goal directed aspects of spatial change. E.g., consider a travelling task from location L_1 to L_2 – Minimally, there are two main closely related aspects to this problem: (a) **Spatial**: The specific sequence of spatial transformations needed in order to achieve a certain desired configuration as well as its *legality*

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or consistency with regard to a set of spatial dynamics. (b) **Causal**: The overall goal or the telic aspect of achieving a desired spatial configuration that dictates why the agent wants to move from L_1 to L_2, which is orthogonal to how precisely to reach location L_2. Our research is driven by the need to treat inferences about the spatial aspects in an integrated manner with inferences about the causal aspects of a system; an endeavour which can be achieved via explicitly relating both the aspects, viz - the effects of actions/occurrences with a set of spatial dynamics. For instance, certain spatial transformations (or their sequences depending on the granularity) could be characterised as being the necessary criteria (pre-condition) for the happening of a particular occurrence. With this as our overall context, the main aim in this paper is to present first ideas on integrating qualitative spatial reasoning with reasoning about the dynamic goal directed & causal aspects of spatial change, an endeavour that can be achieved by formulating a set of causal axioms that relate domain specific *spatial occurrents* in terms of the spatial changes they represent. By spatial occurrents, we refer to events or actions that involve some form of transformation over the spatial structures being modelled. These occurrences can be dealt with at various levels of abstraction, from the most primitive & domain independent to complex, high-level aggregates that are built using the primitives. E.g., in a spatial theory relying solely on mereo-topological relations, a limited notion of occurrences can be defined by exploiting the concept of direct & continuous change between the relations. This abstraction has been used in [6] toward an event based qualitative simulation system. Similarly, a high-level approach would directly model domain specific occurrences on the basis of primitive spatial transitions; e.g., in [3], complex turn actions (such as *turn-left*) have been defined using primitive orientation relations. Our approach is to start with minimal notions about the nature of space & spatial dynamics and investigate the key representational issues that arise whilst dealing with these concepts in conjunction with actions, events & their effects in the context of a general formalism to reason about action & change. Specifically, we perform a naive characterisation of objects based on their physical properties into rigid & non-rigid types. Furthermore, we only use topological knowledge as a spatial metaphor with the continuity [5] of such relations being used to represent spatial dynamics. This approach, we hypothesize, is a necessary pre-requisite toward a general *spatio-teleological theory* that can integrate qualitative spatial reasoning (encompassing multiple aspects of space) with the teleological aspects of spatial change. Such a theory can be used as a basis for modelling dynamic phenomena in a variety of application contexts that require a useful predictive & explanatory capability.

2 Preliminaries - Notation and the Situation Calculus Formalism

The situation calculus as a representational tool for modelling dynamically changing worlds was first introduced in [11]. Some notation follows before the situation calculus formalism used in this paper is presented: Θ_S denotes the set of all spatial theory specific occurrents (spatial transitions). Θ_D denotes the set of all domain specific occurrences including actions (A) and internal (I) & external events (E). Θ denotes the set of all occurrences in the theory. By Φ, we denote the set of all functional & propositional fluents in the dynamic system being modelled. A similar distinction is also made between domain specific (Φ_D) & spatial theory specific (Φ_S) fluents. Finally, we assume that there is a finite number of named occurrences and fluents. The situation calculus formalism used in this paper is essentially a first-order language with the following 5 classes of axioms: **(1)**. Domain specific *action pre-condition axioms* and possibility criteria for various *spatial transitions* definable in the spatial theory are specified using the binary predicate $Poss(\theta,\ s)$, where $\theta\ \in\ \{A\ \cup\ \Theta_S\}$. $Poss$ denotes that occurrence θ possible in situation s. **(2)**. The predicate $occurs(\theta,\ s)$ will be used to denote that all contextual conditions under which event θ can occur are satisfied in a situation s. Here, $\theta\ \in\ \{E\ \cup\ I\}$. **(3)**. A ternary predicate $Holds(\phi,\ v,\ s)$ denoting that fluent ϕ has the value v in situation s. For clarity, we will use it in the following alternative ways: **(a)**. $[\phi(s)\ =\ v]$ or **(b)**. $Holds(\phi,\ v,\ s))$, the latter essentially being the reified version. Finally, a non-determinate situation is expressed in the following manner:$[\phi(s)\ =\ \{\ v_1\ \vee\ v2\ \}]\ \equiv\ [Holds(\phi,\ v_1,\ s) \vee Holds(\phi,\ v_2,\ s)]$ **(4)**. The binary function $Result(\theta,\ s)$, which denotes the unique situation resulting from the happening of occurrence θ in situation s. Here, $\theta\ \in\ \{\Theta_D\ \cup\ \Theta_S\}$. **(5)**. A ternary $Caused(\phi,\ \gamma,\ s)$ predicate, where $\phi\ \in\ \Phi$, denoting that the fluent ϕ is *caused* to take on the value γ in situation s. Note that the $Caused$ predicate is always a direct (direct effects) or indirect link (indirect effects) between fluents & occurrences, an interpretation similar to its proposed use in [9]. For the predicate $Caused$, we need (1a) denoting that if a fluent ϕ is $Caused$ to take the value v in situation s, then ϕ holds the value v in s. (1b) & (2a-2b) denote the unique names & domain closure axioms for truth values & occurrences respectively. We assume that similar axioms for fluents are present as well. We also include a generic *frame axiom* (1c) thereby incorporating the principle of inertia or the non-effects of actions, viz - unless caused otherwise, a fluent's value will necessarily persist.

$$Caused(\phi,\ v,\ s)\ \supset\ Holds(\phi,\ v,\ s) \tag{1a}$$

$$true\ \neq\ false\ \wedge\ (\forall v)\ [v\ =\ true\ \vee\ v\ =\ false] \tag{1b}$$

$$\begin{aligned} Occurs(\theta,\ s)\ \vee\ Poss(\theta,\ s)\ \supset\ \{\neg(\exists v^{'})\ Caused(\phi,\ v^{'},\ Result(\theta,\ s)) \supset \\ Holds(\phi,\ v,\ Result(\theta,\ s))\ \equiv\ Holds(\phi,\ v,\ s)\} \end{aligned} \tag{1c}$$

$$[\theta_i(\boldsymbol{x})\ \neq\ \theta_j(\boldsymbol{y})],\quad where\ i\ and\ j\ are\ different \tag{2a}$$

$$[\theta_i(\boldsymbol{x})\ =\ \theta_j(\boldsymbol{y})]\ \supset\ [\boldsymbol{x}\ =\ \boldsymbol{y}] \tag{2b}$$

3 Actions, Events and Spatial Reasoning - Ontological Issues

3.1 Naive Characterisation of Object Properties

An ontological distinction is made between an object and its *spatial extension*, the latter being denoted by the transfer function $space(object)$. We assume that the spatial extensions are regular regions of space that approximate the object in question. For representational clarity, we define the object-region equivalence axiom in (3a) and simply refer to spatial relationships as holding between *objects* of the domain. In conjunction with a theory of physical objects, the transfer function can be used to make the necessary distinctions. Objects may have varying properties at different times – e.g., take the case of a container filled with water. In this state, one may still drop a metal ball in the water/container. Now assume that in a later situation, the water is frozen and stays that way for eternity. Such changes (e.g., water assuming solid-state) must be reflected as a change of property from a fully-flexible to a rigid object. Another issue is that of the classification of objects into strictly rigid & non-rigid types – when dealing with material (rigid) objects, topological changes can be understood to be the result of motion instead of other possibilities like continuous deformation that are possible with non-rigid objects. However, such a coarse distinction into strictly rigid & non-rigid types is insufficient since many objects exhibit properties of both types. E.g., an object such as a *room* cannot *grow* or *shrink*, but it can *contain* other objects. Therefore, the *room* taken as a whole can neither be classified as being strictly rigid, thereby not allowing interpenetration, nor is it a fully flexible non-rigid object like a water body that can *grow, shrink* or change *shape*. We do not attempt an extensive classification of object categories and the kind of changes permissible therein; rather, only as many distinctions relevant to *growth*, *shrinkage* & *containment* that are necessary in the context of the our example scenario are made. Based on the object properties in (4) and the topological relation ships in the RCC system ($RCC8 \equiv \{dc, ec, po, eq, tpp, ntpp, tpp^{-1}, ntpp^{-1}\}$),domain independent constraints (rigidity (4c), non-rigidity (4d) & semi-rigidity (4e)) can be specified in order to rule out invalid spatial configurations that should not be permitted.

$$(\forall o_1, o_2)(\forall s)\, Holds(top(o_1, o_2), t_{rcc8}, s) \equiv [(\exists r_1, r_2)\, space(o_1) = r_1 \wedge space(o_2) = r_2 \wedge Holds(top(r_1, r_2), t_{rcc8}, s)] \tag{3a}$$

$$rigid(o, s) \equiv [\neg\, allows\_containment(o, s) \wedge \neg\, can\_deform(o, s)] \tag{4a}$$

$$non\_rigid(o, s) \equiv [allows\_containment(o, s) \wedge can\_deform(o, s)] \tag{4b}$$

$$(\forall o, o')(\forall s)\, rigid(o, s) \wedge rigid(o', s) \supset [Holds(top(o, o'), t_{rcc}, s)] \quad where\, t_{rcc} \in \{dc, ec\} \tag{4c}$$

$$(\forall o, o')(\forall s)\, non\_rigid(o, s) \wedge non\_rigid(o', s) \supset [Holds(top(o, o'), t_{rcc8}, s)] \quad where\, t_{rcc8} \in \{dc, ec, po, eq, tpp, ntpp, tpp^{-1}, ntpp^{-1}\} \tag{4d}$$

$$(\forall o, o')(\forall s)\, rigid(o, s) \wedge non\_rigid(o', s) \supset [Holds(top(o, o'), t_{rcc}, s)] \quad where\, t_{rcc} \in \{dc, ec, po, eq, tpp, ntpp\} \tag{4e}$$

3.2 Occurrences - Actions and Events

Ontological distinctions into actions & events only apply at the level of the domain. In the spatial theory, there is only one type of occurrence, viz - a qualitative change as governed by the continuity constraints of the relation space. Within the spatial theory, it is meaningless to ascribe a certain spatial transition as being an event or action; such distinctions demanding a higher level of abstraction. As such, the classification of occurrences into actions & events will only apply at the level of the domain with the present spatial theory dealing only with one type of occurrence, namely a primitive spatial transition definable in it. We represent the same using $tran(t, o_i, o_i)$ (5b), which denotes that o_i & o_j transition to the state of being in relation t. An action is agent-centric and has a non-deterministic will associated with it. Simply, all pre-conditions for a given action may be satisfied and yet the agent may not perform the action. Actions have possibility conditions that are expressed using the $Poss(\theta,\ s)$ predicate. Contrarily, events occur when the criteria for their occurrence holds. The occurrence criteria for an event is governed by its occurrence axiom (5d) where $\gamma(\langle\phi_1,\ldots,\phi_n\rangle\ ,\ s)$ is a situation-dependent first-order formula characterising the necessary & sufficient conditions under which the event will *occur*. With events, a further distinction into external & internal events is made. Events, external to the system, which occur without there being any information as to their occurrence or cause are referred to as *external events*[1]. Events that are internal to the theory and which have associated occurrence criteria are referred to as *internal events*. Internal events are deterministic, i.e., if their corressponding occurrence axiom holds, the event will necessarily occur.

$$\Phi_S \ \equiv\ \{top(o_i,\ o_j),\ allows\_containment(o),\ can\_deform(o)\} \tag{5a}$$

$$\Theta_S \ \equiv\ \{tran(t_1,\ o_i,\ o_j),\ldots,\ tran(t_n,\ o_i,\ o_j)\} \tag{5b}$$

$$\gamma(\langle\phi_1,\ldots,\phi_n\rangle\ ,\ s) \ \equiv\ (\exists\, v_1,\ldots,v_n)[Holds(\phi_1,\ v_1,\ s)\ \wedge\ldots\wedge\ Holds(\phi_n,\ v_n,\ s)] \quad where\ \phi_i\ \in\ \Phi \tag{5c}$$

$$\gamma(\langle\phi_1,\ldots,\phi_n\rangle\ ,\ s)\ \supset\ Occurs(\theta,\ s) \quad where\ \theta\ \in\ \Theta_D \tag{5d}$$

4 A Causal Perspective to QSR in the Situation Calculus

We develop a spatial theory that can be exploited by defining domain specific spatial occurrents in terms of the spatial transformations that are definable in it. An example *delivery scenario* involving the distribution of delivery *objects* to a predefined set of *way-stations* via a delivery *vehicle* will be used throughout. Note that further elaborations of the scenario and the spatial theory are necessary in order to make optimal choices wrt. domain specific parameters such as minimal distance, stops etc.

4.1 Qualitative Spatial Reasoning with Topological Relations

Spatial change is modelled by time-varying topological relationships between objects (i.e., their spatial extensions). This is because topological distinctions are inherently qualitative in nature and they also represent one of the most general ways to characterise space. Precisely, we will use the Region Connection Calculus (RCC) [12] as the spatial theory and provide a causal perspective to it using the Situation Calculus [11]. Our justifications here being that it is necessary to view the problem at such a primitive level before an abstraction involving complex spatial occurrences is directly formalised using the situation (or other) calculus. The RCC-8 system being representative of a general class of similar relational techniques (e.g., JEPD base relations, continuity principle, compositional inference & consistency) that are common in the QSR area – our results can be generalised to cover the class of relational calculi having similar semantics.

[1] Practically, external events can be accounted for in a dynamic planner/controller by continuous interfacing with the external world.

Static Aspect - Composition Table Theorems as State Constraints. Various aspects of the RCC system can be represented using state constraints. E.g., every theorem from the RCC-8 composition table can be modelled as a state constraint thereby resulting in 8×8 constraints of the form in (6a). However, ordinary state constraints pose problems by containing indirect effects in them. In the context of the situation calculus, [9, 10] illustrates the need to distinguish ordinary state constraints from indirect effect yielding or *ramification constraints* – When these are present, it is possible to infer new effect axioms from explicitly formulated effect axioms together with the ramification constraints, i.e., ramification constraints lead to '*unexplained changes*'. This is evident in (6a), where the topological relationships between $o_1,\ o_2$ & o_3 are inter-related and changes in the relationship between any two objects will have an indirect effect on the third one. As such, instead of the ordinary constraint form in (6a), we will use an explicit notion of causality [9] by utilizing the $Caused(\phi,\ \gamma,\ s)$ predicate for the specification of such ramification constraints (6b). Other aspects of the RCC system like JEPD base relations too can be expressed using state constraints. In general, we need a total of n state constraints to express the jointly-exhaustive property of a set of n base relations and $[n(n\ -\ 1)/2)]$ similar constraints to express their pair-wise disjointness. Additionally, other properties such as the symmetry & asymmetry of the base relations too can be expressed using state constraints. Symmetric relations from the RCC-8 set include $dc,\ ec,\ po,\ eq$ whereas $tpp,\ tpp^{-1},\ ntpp,\ ntpp^{-1}$ are asymmetric.

$$(\forall s)\ Holds(top(o_1, o_2), dc, s) \wedge Holds(top(o_2, o_3), ec, s) \supset Holds(top(o_1, o_3), dr, s) \\ \vee\ Holds(top(o_1,\ o_3),\ po,\ s) \vee Holds(top(o_1,\ o_3),\ pp,\ s) \tag{6a}$$

$$(\forall s).\ [Holds(top(o_1,\ o_2),\ dc,\ s) \wedge Holds(top(o_2,\ o_3),\ ec,\ s)] \supset \\ Caused(top(o_1,\ o_3),\ \gamma,\ s) \quad where\ \gamma \equiv [dr \vee po \vee pp] \tag{6b}$$

Dynamic Aspect - Conceptual Neighbourhood Graph as Transitions. In the present spatial theory, the most primitive means of change is an explicit change of topological relationship between 2 objects (their spatial extensions). Let $tran(t_{rcc},\ o_i,\ o_j)$ (5b) denote such a change; read as, o_i and o_j *transition* to a state of being t_{rcc}. (7a) represents the (general) possibility axiom for the said transition. The binary predicate $neighbour(t_{rcc},\ t^{'}_{rcc})$ in (7a) is used to express the consistency of a direct continuous perturbation between 2 relations and is based on the conceptual neighbourhood principle [4], viz - relations t & $t^{'}$ are conceptual neighbours if 2 objects related by t can directly transition to the state of being $t^{'}$ & vice-versa. The conceptual neighbourhood graph for RCC-8 can be used to define a total of 8 transition axioms as in (7a).

Successor State Axioms - Causal Laws of the Spatial Theory. Successor state axioms (SSA) specify the causal laws of the domain – what changes as a result of various occurrences in the system being modelled. Generally, the SSA is based on a *completeness assumption* which essentially means that all possible ways in which the set of fluents may change is explicitly formulated, i.e., there are no indirect effects [13]. The SSA in this section is the one that is derived in the presence of ramification constraints. Recall our use of the ternary causal relation $Caused(\phi,\ \gamma,\ s)$ in (6b) toward the representation of the composition table theorems as state constraints. Precise details notwithstanding, what remains to be done is to minimize the causal relation by circumscribing it (or using some other form of minimization) with the following set of axioms fixed – the foundational (1a-1c) & unique names axioms (2a-2b), the ramification constraints of the form in (6b) and the transition pre-conditions of the form in (7a). The result of minimization is the the *Causation Axiom* in (7b).

$$Poss(tran(t_{rcc},\ o_i,\ o_j),\ s) \equiv [\{space(o_i) = r_i \wedge space(o_j) = r_j\} \wedge \{(\exists\ t'_{rcc})\ Holds(top(\ r_i,\ r_j),\ t'_{rcc},\ s) \wedge neighbour(t_{rcc},\ t'_{rcc})\}] \quad (7a)$$

$$Caused(top(o_i, o_k),\ t_{rcc8},\ s) \equiv \{t_{rcc8} = t_k \wedge (\exists o_j)[Holds(top(o_i,\ o_j),\ t_i,\ s) \wedge Holds(top(o_j,\ o_k),\ t_j,\ s)]\} \quad (7b)$$

$$\begin{aligned} Poss(e, s) \supset\ & [Holds(top(o_i,\ o_j),\ t_{rcc8},\ Result(e,\ s)) \equiv \\ & \{(\forall\ t'_{rcc8})\ top(o_i,\ o_j,\ s) = t_{rcc8} \wedge e \neq tran(t'_{rcc8},\ o_i,\ o_j)\} \vee \\ & \{e = tran(t_{rcc8},\ o_i,\ o_j)\} \vee \\ & \{(\exists o_k, t_i,\ t_j)\ Holds(top(o_i,\ o_k),\ t_i,\ s) \wedge Holds(top(o_k,\ o_j),\ t_j,\ s)\}] \end{aligned} \quad (7c)$$

The causation axiom in (7b) must be integrated with a Pseudo-SSA (PSA) (PSA is SSA without indirect effects) to derive the SSA in (7c). This final result is the SSA-Proper in (7c). In the SSA in (7c), we assume that the final disjunct is the only indirect effect, i.e., we only account for one theorem from the composition table, that too without stating it precisely. In practice, a complete axiomatisation accounting for all disjunctive labels within every non-deterministic composition table entry is required.

$$(\forall s)\ rigid(delivery\_agent,\ s) \quad (8a)$$

$$(\forall d)\ (\forall s)\ rigid(d,\ s) \quad where\ d \in D \quad (8b)$$

$$(\forall s)\ allows\_containment(vehicle,\ s) \wedge \neg\ can\_deform(vehicle,\ s) \quad (8c)$$

$$(\forall w)\ (\forall s)\ allows\_containment(w,\ s) \wedge \neg\ can\_deform(w,\ s),\ where\ w \in W \quad (8d)$$

$$(\forall w_1, w_2)(\forall s)\ [Holds(top(w_1,\ w_2),\ dc,\ s)] \quad (8e)$$

$$(\forall s)\ [\neg Holds(top(delivery\_agent,\ vehicle),\ ntpp,\ s)] \quad (8f)$$

4.2 Integrating Actions and Events with Spatial Reasoning

Domain Theory - Objects and Fluents. The scenario consists of a delivery *agent* & *vehicle*. D is the set of *delivery objects*; W is the set of pre-defined way-stations and O refers to the set of all objects in the domain. Note that ontologically, all elements in O are similar. Depending on their physical attributes, the objects have also been categorised (see 8a-8d) based on the constraints in (4). We also have domain specific constraints (see 8e-8f) specifying (a). way-stations are always *disconnected*, (b). the *delivery_agent* cannot be a *ntpp* of the *vehicle*. Such domain specific knowledge/constraints rule out invalid spatial configurations. Φ_D denotes the following domain specific fluents: $delivery(d,\ w_i,\ w_j)$ – there is a delivery (d) scheduled for pickup from way-station w_i and to be dropped-off at way-station w_j; $signalled(w)$ – a way-station (w)

has been signalled; $halted(w)$ – the delivery vehicle is presently halted at way-station w; and $moving(w_i,\ w_j)$ – the delivery vehicle in currently moving from way-station w_i to way-station w_j. We also have 1 functional fluent, namely $top(o_i,\ o_j)$ (topological relationship between o_i & o_j), with its denotations being closed under the set of RCC-8 base relations. Strictly speaking, this fluent is part of underlying spatial theory with its dynamics being governed by the causal laws defined for the spatial theory. With domain occurrences defined in terms of the spatial configuration of objects, $top(...)$ essentially functions as the bridge between the spatial & causal aspects of the modelled changes.

Domain Occurrences - Actions and Events. The occurrence of actions & events, which constitute the occurrences of the modelled dynamic system, are defined in terms of the spatial configurations which they either cause (effects) or depend on (pre-conditions). E.g., when the delivery vehicle (v) is a non-tangential part of the way-station (w), i.e., $Holds(top(v,\ w),\ s)$, the same is regarded as a occurrence of interest and a named *internal event*, viz - $arrival(w)$, is defined using the $occurs$ predicate in order to reflect the said event. Subsequent to the occurrence of the *arrival* event, the agent may decide to execute a control action (e.g., $halt$ at the way-station iff necessary); the main point here being that it is first necessary to be at the way-station (characterised by a particular spatial pattern) before the *halt* action can be performed.

$$A \equiv \{pickup\_delivery(d,\ w),\ droppoff\_delivery(d,\ w),\ halt(w),\ resume(w)\} \quad (10)$$

$$E \equiv \{schedule\_delivery(d,\ w_i,\ w_j)\}, \quad I \equiv \{signal\_pickup(w),\ arrival(w)\} \quad (11)$$

By Θ_D, we denote the set of all occurrences in the system, viz - actions and internal & external events. We have four actions (A) that the agent may execute. A brief description of these actions is as follows: *pickup_delivery(d, w)* represents the transfer of delivery object d into the way-station; *droppoff_delivery(d, w)* is the transfer of delivery object d from the delivery vehicle to way-station w; *halt(w)* is a control action that will stop the delivery vehicle whereas *resume(w)* is another control action that will resume the vehicle onto its onward course. The scenario has two internal events (I) that occur when certain contextual conditions are met; as explained before, $arrival(w)$ occurs when a spatial configuration denoting the presence of the vehicle inside the way-station is established. Similarly, *signal_pickup(w)* occurs at a way-station every time there is an object scheduled to be picked-up from that way-station. There is only one external event (E) in the system, namely *schedule_delivery* that occurs non-deterministically.

Action Pre-Condition, Occurrence and Effect Axioms. Action pre-conditions (12a) that must hold prior to actual execution are axiomatically specified using the binary predicate $Poss(\theta,\ s)$. Wherever possible, all action pre-conditions are defined in terms the spatial configuration of the domain objects (in addition to non-spatial criteria) that must hold for the action to be executable. Such an explicit link between an action & its spatial pre-requisites can be exploited whilst reasoning about the reachability of certain spatial configurations and the occurrences that either depend on them (pre-conditions) or directly cause them (effect axioms). Effect axioms, modelled using the ternary causal relation $Caused(\phi,\ \gamma,\ s)$, specify what changes directly as a result of the named occurrences. The effect axiom in (12b) denotes that if an occurrence θ either occurs or is possible in situation s, then the fluents $\phi_1, \ldots, \phi_n$ are *caused* to take on the values

$v_1, \ldots, v_n$ respectively. Whenever indirect effects are present, we assume that they are specified using ramification constraints (6b).

$$Poss(\theta,\, s) \,\equiv\, [Holds(\phi_1,\, v_1,\, s) \,\wedge\, \cdots \,\wedge\, Holds(\phi_n,\, v_n,\, s)] \tag{12a}$$

$$\begin{aligned} Occurs(\theta,\, s) \vee Poss(\theta,\, s) \supset [&Caused(\phi_1,\, v_1,\, Result(\theta,\, s)) \wedge \cdots \wedge \\ &Caused(\phi_n,\, v_n,\, Result(\theta,\, s)] \end{aligned} \tag{12b}$$

$$\begin{aligned} Occurs(\theta,\, s) \vee Poss(\theta,\, s) \supset [&Holds(\phi,\, true,\, Result(\theta,\, s)) \equiv \\ &\{\theta = (\theta_1^+ \vee \ldots \vee \theta_n^+)\} \vee \{Caused(\phi,\, true, s)\} \vee \\ &\{Holds(\phi,\, true,\, s) \wedge \theta \neq (\theta_1^- \vee \ldots \vee \theta_n^-)\}] \end{aligned} \tag{12c}$$

Successor State Axioms - Causal Laws of the Domain. Unlike the SSA in (7c), the SSA for the domain theory (12c) universally quantifies over both events & actions. However, the semantics for (12c) are the same, i.e., 'As a result of occurrence θ, fluent ϕ holds in the resulting situation s *iff*–(a). θ explicitly causes ϕ to be $true$, or (b). ϕ is $caused$ to be $true$ as a ramification of something unknown, or (c). ϕ was originally $true$ and continues to be true on the basis of the principle of inertia (1c). This SSA is the reason why direct effects too have been specified using the $Caused$ predicate – On the basis of the effect axioms (12b), the ramification constraints (6b), frame axioms (1c) & minimization of effects (i.e, causal relation), causation axioms encompassing direct & indirect effects are obtained for the derivation of the SSA's [9, 10].

$$\Phi \,\equiv\, [Holds(top(o_1,\, o_2),\, t_1,\, s) \,\wedge\, Holds(top(o_2,\, o_3),\, t_2,\, s)] \tag{13a}$$

$$\begin{aligned} &\Phi \wedge RCC8_{CT} \vdash \Phi^{'} \\ where\, \Phi^{'} \equiv\, &[Holds(top(o_1,\, o_2),\, t_1,\, s) \wedge Holds(top(o_2,\, o_3),\, t_2,\, s) \wedge \\ &Holds(top(o_1,\, o_3),\, t^{'},\, s)] \end{aligned} \tag{13b}$$

Initial Situation. A description of initial fluent values when no occurrences have happened is needed: For spatial fluents, there exist 2 classes: those which model the spatial relationship between objects (e.g., top) & those which characterise the dynamic object properties (e.g, $allows\_containment$). The case for non-spatial fluents (e.g., $moving$) and spatial fluents characterising object properties is trivial and will be omitted. As for the fluents encompassing spatial relationships, the initial situation description involving n domain objects requires a complete $n - clique$ description with $[n(n\,-1)/2]$ spatial relationships. However, relationships between some objects may be omitted in which case a complete initial situation description (with disjunctive labels) can be derived on the basis of the RCC-8 composition table. As an example, consider the simplest case involving 3 objects in (13): Φ denotes a partial description involving the 3 objects, viz - the relationship between objects o_1 and o_3 is unknown. Given Φ, $\Phi^{'}$ can be monotonically derived ($\vdash$) on the basis of Φ and the RCC-8 compositional table constraints (RCC_{CT})[2]. Here, Φ^1 is a monotonic extension of Φ in the sense that whilst new information is conjoined with Φ, none of the existing spatial knowledge is invalidated.

[2] Cui et al. [2] propose to maintain only those state descriptions that arise from pairs of objects in the initial situation. This will be the default behaviour when '$\vdash$' is not applied.

5 Discussion and Outlook

Our main interest lies in the development of a set of spatial primitives/patterns (and their combinations) that are necessary to satisfy a predictive & explanatory function for a particular domain. A corpus of spatial primitives (akin to a library of spatial dynamics) can be used to perform such functions within a domain; the main objective here being that it should be possible for domain modellers to use the underlying spatial theory for different application specific purposes. Specifically, the extensions can be used to model computational tasks such as: (a) *Spatial Planning*: Derive a sequence of spatial actions that will fulfill a desired objective. In other words, "How do we transform one spatial configuration into another?". Note that an online (incremental) plan generation approach that incorporates dynamically available information (e.g., an agent may have sensing capabilities) is more powerful in comparision to offline or a static one. As such, the theory developed in this paper can be implemented in the context of existing situation calculus based languages (e.g., [8]) that provide such capabilities. (b) *Causal Explanation*: Given a set temporally ordered snap-shots, the aim is to extract a explanatory causal model in terms of spatial occurrents from the given spatio-temporal data; the main objective here being, "What occurrences may have caused a particular spatial configuration, or a series of temporally ordered configurations?". Furthermore, it must be emphasized that explanation can only be provided for qualitative changes as reflected by the underlying relation space. For instance, in the absence of distance information, time varying distances between two *disconnected* objects will be not reflected within the theory. (c) *Spatial Projection & Interpolation*: Given an initial situation, predict all possible evolutions of the system or interpolate missing spatial knowledge between two temporal snap-shots of a changing system. Reachability of a particular situation could be characterised by the happening of one or more occurrences or by some spatial configuration of the objects in the domain.

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