A General Framework Based on Dynamic Constraints for the Enrichment of a Topological Theory of Spatial Simulation

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Abstract. Qualitative spatial representation and reasoning has emerged as a major sub-field of AI in the past decade. An important research problem within the field is that of integrated reasoning about various spatial aspects such as distance, size, topology etc - an important application here being the qualitative simulation of physical processes. Approaches based on topology alone fail to provide an explicit account of other important aspects of spatial change thereby also not utilizing dynamically available information pertaining to them. Our work in this paper is based on the idea that a general theory of spatial simulation based on topological changes alone can be enriched by the inclusion of sub-theories relevant to other aspects of spatial change. We propose a general framework consisting of dynamic constraints for the enrichment of a topological theory of spatial changes. We propose the utilisation of such dynamic constraints for the incorporation of dynamically available information relevant to various aspects of space thereby making that aspect explicit in the theory. As an example of the proposed approach, we integrate dynamically available information pertaining to motion and size with the topological theory of RCC-8 using our framework of dynamic constraints.

1 Introduction

Qualitative reasoning about space and time - *reasoning at the human level* - promises to become a fundamental aspect of future systems that will accompany us in daily activities [1]. Indeed, the past decade has witnessed the growth of research in qualitative spatio-temporal representation and reasoning and its emergence as an active sub-field of AI [2, 3]. Dealing with uncertainty in space-time has yielded a much more developed research trend [1] – that of *spatial envisionment* or motion extrapolation, an important application of such envisionment being the *qualitative simulation of physical processes*

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[4–7]. The aim here is to simulate the spatio-temporal behavior of, in general, solid objects or more simply to predict the result of motion. Within the context of qualitative representational formalisms, the most commonly applied technique is that of a *Continuity Network* or *Conceptual Neighbourhood Diagram* (CND), a term originating from [8]. Given a relational theory for representing spatial information, a CND associated with that theory is essentially a graph that tries to systematically capture the continuity of a change of relation between spatial objects – with the nodes representing primitive relations from the theory and edges representing direct, continuous transitions between them. For example, Fig. 1 shows the direct transitions possible for a topological theory of spatial changes, namely the Region Connection Calculus (RCC) [9]. It consists of the following set of eight jointly exhaustive and pair-wise disjoint (JEPD) relations: DC – *disconnected*, EC – *externally connected*, PO – *partially overlapping*, EQ – *equal*, and TPP – *tangential proper-part* & NTPP – *non-tangential proper-part* along with their respective inverses. In essence, two spatial relations are neighbours in a CND if a continuous change can yield a direct transition from one relation (node) to the other. This idea of the continuity of spatial change, with respect to a background theory for representing spatial information, inherent in the CND has been exploited towards qualitative reasoning about spatial changes in [5, 10, 11].

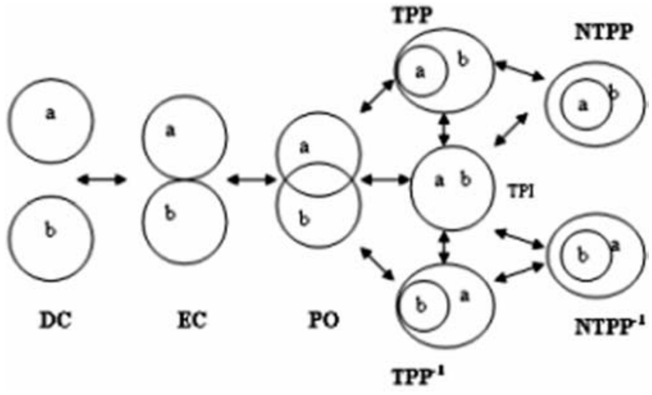

Fig. 1. Direct Topological Transitions for RCC8 Relations

Notice that the topological transitions in Fig. 1 are implicitly subject to the satisfaction of various constraints over the spatial objects involved. For example, consider a scenario pertaining to the motion of objects in a $n - dimensional$ space: assume two disconnected (DC) regions, atleast one of them moving towards the other. If they collide, we have EC holding between them. If they bounce or brush off, we return to DC. Otherwise, they go through each other and we have PO. At this point, we have four possibilities for the next spatial relation: EC, EQ, TPP or TPP^{-1}. Exactly which transition happens next depends upon the *relative motion and sizes* of the objects. Furthermore, each of these constraints can be classified on the basis of the spatial aspect that it deals with; the spatial aspect that has been left implicit in the theory of topological changes.

For example, consider motion that has not been treated explicitly in the transition network of RCC-8 [5]. Likewise, relative size constraints between regions too have not been accounted for in the direct topological transitions between RCC-8 relations. This paper contains two main contributions: **(a)** We propose a general framework consisting of dynamic constraints for the enrichment of a topological theory of spatial changes. We propose the utilisation of such dynamic constraints for the incorporation of dynamically available information relevant to various aspects of space such as distance, size, position etc, thereby making that aspect explicit in the theory. **(b)** Within the context of our framework, we incorporate motion and size constraints into the topological theory of spatial changes based on RCC. Motion in this paper is based on a simple notion of time-varying distances whereas size is based on primitive comparison relations between the $n - dimensional$ measures of the regions involved.

2 Sub-theories Based on Dynamic Constraints

2.1 The General Framework

We propose a general framework consisting of a suite of sub-theories, each encompassing a different aspect of spatial change such as distance, size, motion etc, within a theory of spatial changes that is based on topology alone. A constraint (see Def. 1) is dynamic in the sense that it can only be satisfied when dynamic information, pertaining to distance, size, position etc, is available.

Definition 1. *A* ***Dynamic Constraint*** *within our framework is a n-ary temporal predicate that may or may not hold a certain point in time. Dynamic constraints are of the form* $\phi(arg_1,\ \ldots,\ arg_n)$ *and every such constraint has a implicit temporal argument* t*. A* ***Dynamic Constraint Set****, denoted as* $\Theta_{subtheory}$*, is a set consisting of a finite number of such dynamic constraints.*

All dynamic constraints in this paper are binary (temporal argument not included) and will be of the following form: $\phi(arg_1,\ arg_2,\ t)$. For clarity, we may omit the temporal argument in certain cases. Every dynamic constraint set $\Theta_{subtheory}$ characterizes a sub-theory pertaining to a certain aspect of spatial change that needs to be explicitly modelled so as to complement the basic theory of topological changes. As an example consider the sets $\Theta_M\ and\ \Theta_S$ in section 3, which denote the class of motion and size constraints that have been included within the theory of topological changes. The inclusion of these constraints has the effect of providing an explicit account of the the relative motion and sizes of the objects involved.

Definition 2. *A* ***Dynamic Constraint Suite****, denoted by* Σ*, is the set consisting of all dynamic constraint sets to be included as sub-theories with the framework. More formally,* $\Sigma\ \equiv\ \{\Theta_1,\ \Theta_2,\ \ldots\ \Theta_n\}$*, where each of the* Θ_i *represents dynamic constraints relevant to a certain aspect of spatial change being modelled within the framework.*

We refer to a collection of such sub-theories as a dynamic constraint suite. Important, from a computational viewpoint, is the idea of the satisfiability of a set of dynamic constraints $\Theta_{subtheory}$, or $\Sigma - Satisfiability$ in general. Within the context of our framework, the same has been stated more precisely in Def. 3 below:

Definition 3. ***Satisfiability*** *- A set of dynamic constraints, given by $\Theta_{subtheory}$, on a transition between two qualitative states is satisfiable iff $\forall\phi$ $holds(\phi,\ t)$, where $\phi \in \Theta_i$. Here, $holds(\phi,\ t)$ should be interpreted as the successful validation of the constraint ϕ based on the dynamically available information at time point t. Likewise, satisfiability of all dynamic constraints within our framework Σ is defined to be the satisfiability of every dynamic constraint set $\Theta_{subtheory}$ contained therein.*

2.2 Constraints on Topological Transitions

Def. 1 – 3, which provide a general representational framework for the specification of the dynamic constraints, will be used to formalise the incorporation of dynamic constraints within the theory of topological changes. We introduce the idea of a ***transition constraint*** - *A transition constraint is essentially a dynamic constraint, but one that is imposed on a certain transition between two topological relations*. The satisfiability criteria for a transition constraint is similar to that of a dynamic constraint expressed in **Def.** 3. Again, intuitively the satisfiability entails that a certain relational transition with respect to the base topological theory is consistent with the dynamically available (for e.g. sensory) information relevant to the respective spatial aspect that the constraint is supposed to model. A more precise formulation is presented in the following.

Let **R** denote the set of JEPD relations included in the RCC-8 theory – **R** $\equiv \{dc,\ ec,\ po,\ eq,\ tpp,\ ntpp,\ tpp^{-1},\ ntpp^{-1}\}$. Let **T** be a set consisting of ordered pairs of the form $< r_i,\ r_j >$ (a binary relation on **R**). For clarity, we refer to such a ordered pair using the predicate $trans(r_i,\ r_j)$. Its intended semantics is that a direct, continuous transition from $r_i\ to\ r_j$ is legal (though not necessarily possible). Note that the set **T** can be easily defined by simple enumeration on the basis of the transition network of RCC-8 shown in Fig. 1. Also, the scope of the universally quantified region variables $x\ and\ y$ in the definition **P** below is understood to be the subset of regions that are related by r_i at time point t_1. The following is the formal definition for a transition constraint:

(P) $poss(< r_i,\ r_j >,\ t) \equiv [(\forall x,\ y)\ (\exists\, t_1\ t_2)]$
$(t_1 < t_2) \wedge (t_2 < t) \wedge holds(r_i,\ x,\ y,\ t_1) \wedge \phi_1(x,\ y,\ t_2) \wedge \ldots \wedge \phi_k(x,\ y,\ t_2).$
where $< r_i,\ r_j > \ \in$ **T**, $and\ \phi_i\ \ldots\ \phi_k \in \Theta\ from\ \Sigma$

The intended interpretation for **P** defined above is that a direct, continuous and legal transition from $r_i\ to\ r_j$ is *possible* at time point t if regions $x\ and\ y$ have the relation r_i between them at some time-point t_1 before t and that at other time-point t_2 between $t_1\ and\ t$, the dynamic constraints imposed on the the legal transition $trans(r_i,\ r_j)$ can be *satisfied* on the basis of the dynamically available information pertaining to the spatial aspect being modelled by the respective constraint.

2.3 Minimal and Σ-Consistency

The idea of ***minimal and*** Σ***-consistency*** has been informally defined below in Def. 4. Minimal consistency essentially entails that absolutely no transition constraints have been imposed on the direct topological changes possible between RCC-8 relations. It can be easily verified that the concept of $\Sigma - Consistency$ is much more richer than the minimal/general account of spatial changes provided by the RCC-8 direct topological transitions. This is because depending on the number and nature of the additional sub-theories included within Σ, additional information that is dynamically available will be utilised whilst simulating the physical system.

Definition 4. ***Consistency of an Envisionment*** *- An envisionment*[1] *of a physical system, either partial or complete, is* ***minimally consistent*** *iff the transitions between qualitative states (topological relations between objects) of the system are solely based on the direct topological transitions possible wrt the RCC-8 conceptual neighbourhood diagram. Similarly, the envisionment is* ***maximally consistent***[2]*, also referred to as '*Σ***-Consistent****', iff the transitions between the states* satisfy *every set of constraints included in the* Σ*, in addition to being consistent with the RCC-8 CND.*

A formal account of the Σ-Consistency of transitions between qualitative states or a complete envisionment of a physical system can be derived by generalizing the definition of a transition constraint **P** given in section 2.2. Precisely, the definition of $poss(r_i,\ r_j)$ entails the consistency of a single transition, for e.g., from DC to EC, involving two objects. This definition needs to be generalised for a arbitrary chain of *legal transitions* $r_0,\ r_1,\ \ldots,\ r_n$ involving more than two objects. Such a chain of transitions is $\Sigma - Consistent$ *iff* $poss(r_i,\ r_{i+1})$ is true for all $< r_i,\ r_{i+1} >$. These definitions can easily be extended to handle domains consisting of a arbitrary number of objects. However, the definitions already presented are sufficient for the purposes of this paper.

3 Motion and Size Constraints on Topological Transitions

In this section, we illustrate how the formal framework developed in Section 2 can be applied - we illustrate the incorporation of dynamically available information pertaining to the relative movement and sizes of objects. Note that this paper does not attempt to provide a full-scale treatment of motion or relative sizes of spatial objects. Our use of motion and size constraints in this section is only exemplary of the manner in which the proposed framework involving dynamic constraints is to be utilised.

(A1) $DC(x,\ y) \equiv_{def}\ dist(x,\ y)\ >\ \delta$
(A2) $EC(x,\ y) \equiv_{def}\ dist(x,\ y)\ \leq\ \delta\ \wedge dist(x,\ y)\ \neq 0$
(A3) $Co(x,\ y) \equiv_{def}\ dist(x,\ y)\ =\ 0$

We define a class of motion constraints Θ_M (see Def. 5) that are based on a simple notion of time-varying distances between objects. These constraints are in turn based on three dyadic relations, DC - disconnected, EC - externally connected and CO - coalesce, between pairs of regions expressed using the function $dist(x,\ y)$ (see A1-A3). For e.g., two regions are *disconnected* if the degree of displacement between them is greater a

[1] An envisionment is a temporal partial ordering of all the qualitative states a modelled physical system can evolve into given some indexed/initial state [12]

[2] Of Course, within the context of our theory of dynamic constraints

certain δ, where δ is a domain dependent parameter. We sometimes use the primitive function $dist(x, y)$ with a temporal argument, as in $dist(x, y, t)$, to denote the distance between x and y at time-point t. The distance function can be intuitively understood as the size of the shortest line connecting any two points in the two region boundaries. The concept of distance should be understood as a qualitative notion of displacement, i.e., the accurate measurements are not important [13], but how the distance between regions varies with time so as to capture their relative movement.

Definition 5. *The class of* ***motion constraints*** Θ_M *includes dynamic constraints of the form discussed in Def. 1. Specifically,* Θ_M *consists of the five motion relations based on the primitive notion of distance between two regions that are defined axiomatically in C1-C5:* $\Theta_M \equiv \{\phi_{ap}, \phi_{re}, \phi_{sp}, \phi_{co}, \phi_{st}\}$.

(C1) $\phi_{ap}(x, y, t) \equiv_{def} [(\exists\, t_1\, t_2)]\, (t_1 < t) \wedge (t < t_2) \wedge holds(DC, x, y, t_1) \wedge \neg\, holds(CO, x, y, t_2) \wedge dist(x, y, t_1) > dist(x, y, t_2)$

(C2) $\phi_{re}(x, y, t) \equiv_{def} [(\exists\, t_1\, t_2)]\, (t_1 < t) \wedge (t < t_2) \wedge [holds(DC, x, y, t_1) \vee holds(EC, x, y, t_1)] \wedge dist(x, y, t_1) < dist(x, y, t_2)$

(C3) $\phi_{sp}(x, y, t) \equiv_{def} [(\exists\, t_1\, t_2)]\, (t_1 < t) \wedge (t < t_2) \wedge [holds(DC, x, y, t_2) \vee holds(EC, x, y, t_2)] \wedge holds(CO, x, y, t_1)$

(C4) $\phi_{co}(x, y, t) \equiv_{def} [(\exists\, t_1\, t_2)]\, (t_1 < t) \wedge (t < t_2) \wedge [holds(DC, x, y, t_1) \vee holds(EC, x, y, t_1)] \wedge holds(CO, x, y, t_2)$

(C5) $\phi_{st}(x, y, t) \equiv_{def} [(\exists\, t_1\, t_2)]\, (t_1 < t) \wedge (t < t_2) \wedge dist(x, y, t_1) = dist(x, y, t_2)$

The following interpretation holds for the motion constraints defined in Θ_M: $\phi_{ap}(x, y, t)$ – x & y are approaching each other at time-point t, $\phi_{re}(x, y, t)$ – x & y are receding from each other, $\phi_{sp}(x, y, t)$ – x & y are splitting, $\phi_{co}(x, y, t)$ – x & y are coalesce, and $\phi_{st}(x, y, t)$ – x & y are static.

We also define define a class of size constraints Θ_S (see Def. 6) that are based on primitive comparison relations between the sizes of the regions involved. Similar to [14], we assume that the size of a n-dimensional region corresponds to its n-dimensional measure. For example, the size of a sphere in R^3 corresponds to its volume. The function $size(x)$ will be used to denote the size of an region given by x.

Definition 6. *The class of* ***size constraints*** Θ_S *includes the following dynamic constraints that relate the size of two regions at a certain time point:* $\Theta_S \equiv \{\phi_<, \phi_>, \phi_\le, \phi_\ge, \phi_=\}$. *Each of the size constraints is of the form* $\phi_{size\_rel}(x, y, t)$*, where* $\phi_{size\_rel} \in \Theta_S$. *For notational convenience, the interpretation of* $\phi_{size\_rel}(x, y, t)$ *is explained with an example:* $\phi_<(x, y, t)$ *should be interpreted as '*$size(x) < size(y)$*' at time t, where* $size(x)$ *is the size of region* x*, and* $size(y)$ *is the size of region* y*.*

We now have a dynamic constraint suite $\Sigma \equiv \{\Theta_M, \Theta_S\}$. Dynamic constraints that make up the theory Σ can now be used for the definition of transition constraints, i.e., dynamic constraints imposed on the topological transitions (see section 2.2). The precise representational form of the transition constraints definable within the context of our theory Σ will follow the generic definition in **P**.

(T1) $poss(trans(dc,\ ec),\ t)\ \equiv [(\forall x,\ y)\ (\exists\, t_1\, t_2)]$
$(t_1\ <\ t_2)\ \wedge\ (t_2\ <\ t)\ \wedge\ holds(dc,\ x,\ y),\ t_1)\ \wedge\ [\phi_{ap}(x,\ y,\ t_2)]$

(T2) $poss(trans(ec,\ dc),\ t)\ \equiv [(\forall x,\ y)\ (\exists\, t_1\, t_2)]$
$(t_1\ <\ t_2)\ \wedge\ (t_2\ <\ t)\ \wedge\ holds(dc,\ x,\ y,\ t_1)\ \wedge\ [\phi_{re}(x,\ y,\ t_2)]$

(T3) $poss(trans(po,\ eq),\ t))\ \equiv [(\forall x,\ y)\ (\exists\, t_1\, t_2)]$
$(t_1\ <\ t_2) \wedge (t_2\ <\ t) \wedge holds(po,\ x,\ y,\ t_1) \wedge [\phi_{co}(x,\ y,\ t_2) \wedge \phi_{=}(x,\ y,\ t_2)]$

(T4) $poss(trans(po,\ tpp),\ t))\ \equiv [(\forall x,\ y)\ (\exists\, t_1\, t_2)]\ (t_1\ <\ t_2)\ \wedge\ (t_2\ <\ t)$
$\wedge\, holds(po,\ x,\ y,\ t_1)\ \wedge\ [\neg\phi_{co}(x,\ y,\ t_2)\ \wedge\ \phi_{<}(x,\ y,\ t_2)\ \wedge\ \neg\phi_{sp}(x,\ y,\ t_2)]$

For parsimony of space, we only illustrate a subset of the transition constraints that are definable by way of the axioms **T1-T4** above. The dynamic constraints from our framework that are imposed on the transitions are highlighted in the brackets. An an example, consider transition constraint **T4** $- -poss(trans(po,\ tpp),\ t))$. It enforces the condition that a legal transition between two regions from po to tpp is *possible* at time-point t iff at some time-point $t_1\ <\ t$, $x\ and\ y$ were po at t_1, and at another time-point t_2, such that $t_1\ <\ t_2\ <\ t$, the dynamic constraints imposed on the transition are *satisfiable*. Notice how this simple axiom integrates three differing aspects of spatial change – topology, motion & size. A similar discussion applies for other transition constraints as well and will been left out.

4 Discussion and Further Work

The overall context of our work is centered around the idea of an envisionment based qualitative simulation program [12]. The work in [5] utilizes the the same approach and is based on a logical theory of topological changes alone. Although their work utilizes constraints, they are used in a static and domain dependent manner in the form of inter & intra-state constraints. Motion, size or any other aspect of space is not treated explicitly and dynamically available information has not been utilized. Using our approach, the specification of some of the domain specific constraints becomes redundant – for r.g., Consider two regions a and b, such that $size(a)\ <\ size(b)$ at all times. Since a is smaller than b, one constraint that always need to be maintained (an inter-state constraint) is that b can never be a tangential or non-tangential proper-part of a. Since size constraints are explicitly accounted for in the form of transition constraints in our theory, no domain specific constraint to the effect needs to be specified by the domain modeller.

The formulation of motion constraints in Section 3 is based on the idea of defining connectedness between two objects using the primitive notion of displacement between them. This idea arises from [13], though in a somewhat different context. Using this formulation, the relation of *partial overlap* (PO) between two regions cannot be expressed when connectedness is defined using the primitive the notion of distance (see $A1 - A3$). As such, dynamic constraints on $trans(ec,\ po)$ or $trans(po,\ ec)$ are not definable. This, however, reflects the lack of expressivity of the manner in which motion is defined. Dealing with such inadequacies or providing a full-scale treatment of motion, which is beyond the scope of our research interests, has already been treated elsewhere [11].

Non-determinism or ambiguity is represented in RCC's transition network in the form of multiple edges emanating from one node to other. This ambiguity can be reduced (atleast partially) by making use of dynamically available of information pertaining to relative motion and sizes – dynamic constraints can be used to restrict the simulation to a certain region of the state space. Ofcourse, this assumes the presence of a sub-theory for motion, sizes etc within the spatial simulation framework to complement the theory of spatial changes based on topology alone. With this motivation, our long-term research goal is to investigate integrated ways to represent & reason about various spatial aspects such as distance, topology, size, orientation, motion etc. We consider a dynamic constraint based approach to be a suitable candidate contributing toward such a goal. The main idea here being that of utilising the dynamically available information relevant to various spatial aspects whilst simulating a system.

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