A Causal Approach for Modelling Spatial Dynamics*
A Preliminary Report

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Abstract

We propose a causal approach, involving events identified by their causes and effects, for the modelling of spatial dynamics. The suitability of situation calculus as a high-level formalism for representing and reasoning about spatial dynamics is explored and the causal framework is formalised using the same. A systematic illustration of the manner in which various aspects of axiomatic qualitative spatial calculi may be represented within the proposed causal framework is presented. The main advantage of this approach is that based on the structure and semantics of the calculus, computational tasks such as planning, projection and explanation can be directly exploited. Within the specialised spatial reasoning domain, these translate to spatial planning/re-configuration, simulation and causal explanation (i.e., inferring cause from observations). Furthermore, given the qualitative nature of the spatial theory and the non-monotonic reasoning capability within the formalism, the approach is also better suited at representing human-like abilities of common-sense reasoning with incomplete information. The main hypothesis underlying our approach is that an alternate causal perspective of existing qualitative spatial calculi using high-level tools such as the situation calculus is essential for their utilisation in diverse application domains such as intelligent systems, cognitive robotics and event-based and Temporal-GIS.

1 Motivation

Most research in qualitative spatial reasoning has focussed on the development of spatial calculi that are representative of distinct spatial domains – mereotopology [Randell et al. 1992], orientation [Freksa 1992, Moratz et al. 2000], distance [Hernandez et al. 1995], cardinal directions [Ligozat 1998] etc. Furthermore, there has been considerable progress toward efficient computational mechanisms for reasoning within the respective spatial domains (see [Cohn and Hazarika 2001a] for a complete overview). However, relatively little work has explicitly addressed the need to develop or exploit existing representational apparatus that will facilitate the use these spatial calculi in realistic application domains – alternate views of existing spatial reasoning techniques are essential if spatial calculi encompassing unique or integrated spatial domains are to be utilised in realistic application scenarios, e.g., as control mechanisms in robotic software/intelligent systems, representation of human-like spatial reasoning or decision-making abilities in real and/or simulated environments or even as explanatory models within event-based and temporal-GIS (TGIS) applications where representing and reasoning about dynamic geospatial phenomena is of utmost importance. There are a wide range of formalisms that have been developed for representing and reasoning about dynamically changing environments (e.g., situation calculus [McCarthy and Hayes 1969], events calculus [Kowalski and Sergot 1986]). The utility of such higher level representational formalisms (involving reasoning about actions and change) for the modelling of spatial dynamics cannot be taken granted – rather fundamental problems (e.g., Frame, Ramification, Qualification [Shanahan 1997]) relevant to modelling changing environments have been thoroughly investigated in the context of the class of formalisms aforementioned. This has resulted in several non-monotonic extensions to classical symbolic approaches that are better suited for representing human-like abilities of common-sense reasoning with incomplete information. Furthermore, the issue of concurrent and continuous phenomena, which manifest themselves even in the simplest of dynamic domains, has been rigorously investigated in the context of the class of formalisms developed within the area of reasoning about actions and change. This is especially useful for modelling of spatial dynamics, considering that the issue of concurrent spatial changes has not been addressed within the specialised spatial reasoning domain.

Based on the situation calculus formalism, we propose a causal approach for the representation of spatial dynamics. The approach utilises an explicit notion of causality involving events that are identified by their causes and effects. Several distinctions of occurrences into internal and external events and actions are applicable. However, in this paper, only a limited notion of an event based on primitive spatial transitions definable within in a qualitative spatial theory is used. For the spatial part of the theory, we operate within a purely region-

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based framework that is suitable for fine-scale analysis with primitive objects or macro-level analysis with aggregates of entities that have a well-defined spatiality. Additionally, we perform a naive characterisation of objects based on their dynamic physical properties in order to constrain the possible changes they may undergo in a domain-independent manner. Most importantly, we systematically illustrate how various aspects of qualitative spatial calculi can be represented within a causal framework. The approach is general enough so as to encompass the wide range of axiomatic calculi, relevant to differing spatial domains, that are based on similar semantics – a finite set of jointly exhaustive and pair-wise disjoint relations, compositional reasoning and consistency maintenance and the continuity of the underlying relation space. From the proposed causal framework and the structure and semantics of the situation calculus, several computational tasks such as explanation, planning and projection directly follow. On the basis of these computational tasks, the utilisation of the proposed causal framework is proposed for applications in the areas of cognitive robotics and event-based GIS.

2 A Causal Approach for Modelling Spatial Dynamics

A Causal Approach

We propose the utilisation of a causal approach, based on the situation calculus formalism, for representing and reasoning about spatial dynamics. The rich ontology of the situation calculus formalism (events, actions and a general mechanism to formalise change) and the modelling of spatial dynamics using it lends itself to useful computational tasks that are applicable in a diverse range of applications – based on the structure and semantics of the calculus, computational tasks such as causal explanation, spatial planning and spatial simulation directly follow. These computational tasks are useful for modelling and analysis in a wide range of geospatial phenomena or even in a real-time system involving the surveillance of spatial scenes where certain observable spatial changes can be directly linked to known actions or events. Furthermore, the approach also accounts for the teleological/purpose-directed aspects of spatial change – it is possible to infer purpose from observed change or prescribe change (e.g., spatial re-configuration or planning) based on purpose, thereby serving as a goal-directed control mechanism in intelligent robotic applications. Note however inferring purpose from change/observations or prescribing change based on purpose is only possible if there is indeed a teleological aspect to the spatial changes being modelled per se. For instance, whereas there can be a telic/purposive aspect to the sequence of spatial changes determined by the *turn-actions* that a *vehicle* may undertake whilst following a route description, the same may not be applicable in a situation such as the follows: '*The village was washed away in the tsunami*', where although *causation* is applicable, but without a telic aspect. We hypothesize that teleology necessarily involves causation (or a causal specification) comprising of purpose/goal directed occurrences (that are typically actions) with well-defined pre-conditions and effects, whereas causality by itself does not necessarily entail purpose/goal-directedness[1]

Events within a Causal Framework

The ontological status of events has been an issue of much discussion and debate among philosophers [Davidson 1969, Kim 1976, Pianesi and Varzi 2000, Quine 1960]. According to Quine [1960], events are to be regarded (in a manner similar to objects) as spatio-temporal regions with at most one event occupying a given spatio-temporal region of space. This position has been promoted by several researchers in the qualitative spatio-temporal reasoning domain toward the development of mereo-topological, spatio-temporal theories of qualitative spatial change [Hazarika and Cohn 2001, Muller 1998a;b] and the proposed application of such spatio-temporal frameworks in varied application contexts such as qualitative (robotic) localization [Cohn and Hazarika 2001b] and temporal-GIS [Cohn and Hazarika 2001c]. At the heart of these spatio-temporal theories lies the premise that objects (continuants) and events (occurrents) are not to be distinguished and both be regarded spatio-temporal regions of space, i.e., space-time histories of events and objects be accorded a primitive ontological status within the theory. Indeed, the original region-based calculus in [Clarke 1991], on which the mereo-topological spatio-temporal theories are founded, too had a spatio-temporal interpretation.

The notion of events that is applicable within our framework is causal in nature and is aimed at characterising explicit causal and (if applicable) teleological accounts of the evolution of a process. This view is based on an alternate view of events, where events are identified according to their causes and effects [Davidson 1969]. As Davidson elaborates: '*Events have a unique position in the framework of causal relations between events in somewhat the same way objects have a unique position in the spatial framework of objects*' [Davidson 1969; pg. 179]. Events within a causal framework (i.e., events identified by their causes and effects) may be interpreted differently depending on the problem being addressed. In general, the following distinctions are applicable:

1. Internal Events: Events that are internal to the system being modelled and which have a associated occurrence criteria are referred to as *internal events*. Internal events are deterministic in the sense that if the occurrence criteria for an internal event is satisfied, the event will necessarily occur.

2. External events: Events that are external to the system and which occur arbitrarily are referred to as *external events*. By arbitrary, we mean that unlike internal events, occurrence criteria for these events are not available. An as example, consider a simulation of the queue at a bank teller. As far as the simulation is concerned, an event characterised by the arrival of a new customer at the queue is something external to the simulation of the queue and should be regarded as being

[1] An in-depth analysis of these concepts in the context of modelling spatial dynamics for a specialised domain will be the object of another research, much beyond the scope of the our present aims.

non-deterministic; the only surety from a simulation perspective being that at some point, the said event will certainly occur. Note that the event may not be arbitrary in actuality. However, in so far as theory and the simulation model is concerned, its occurrence can be treated as such. Practically, external events can be accounted for within the context of a dynamic planner/controller where the system can continuously interface with the external world to poll for the occurrence of such events.

3. Non-deterministic Events or Actions: Actions are agent-centric (i.e., performed by an agent) and are therefore, by definition volitional or have a non-deterministic will associated with them. Instead of occurrence criteria, actions are governed by possibility conditions subject to which they may or may not happen. Simply, all pre-conditions for a given action may be satisfied and yet the agent may not perform the action. The distinction into actions is mainly applicable in scenarios where spatial reasoning abilities of real or simulated agents are being modelled, e.g., robotic control software.

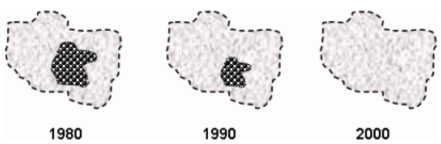

Fig. 1: Shrinkage and Disappearance

Henceforth, we refer to internal & external events and actions as occurrences. In the specialised spatial reasoning domain, occurrences may be defined at two levels: (1) On the basis of a typology of the fundamental spatial changes, which the primitive entities within the spatial theory may undergo, e.g., *growth, shrinkage, splitting, merging, appearance, disappearance, rotation and movement* [Claramunt and Thériault 1995]. At this level, the only identifiable notion of an occurrence is that of a qualitative spatial transition that the primitive objects in the theory undergo. (2) Domain specific spatial occurrences (events or actions) that have (explicitly) identifiable occurrence criteria and effects that can be defined in terms of the fundamental typology of spatial change. For instance, in the example in Fig. 1, we can clearly see that the contained/smaller region has continued to *shrink* over a 3 decade period, eventually *disappearing* altogether in the year 2000. In so far as a general theory of space or spatial dynamics is concerned, the only applicable/identifiable notion of *events* will be based on a primitive taxonomy of spatial change, i.e., in the example under consideration, the only interesting or identifiable events are *shrinkage* and *disappearance*. However, at a domain specific level, the observed phenomena can be causally related to *deforestation*, *fire* or other events. As such, at the domain-specific level, the following notion of a ***spatial occurrence*** is applicable – 'spatial occurrences are either *events* or *actions* with explicitly specifiable *occurrence criteria* or *pre-conditions* respectively and *effects* that may be defined in terms of a domain independent taxonomy of spatial change that is native to a spatial theory. For example, a certain spatial event may *cause* a region to *split* into two or make it *grow/shrink*'. Likewise, a spatial (control) action, e.g., *turn-left*, will have the effect of changing the orientation of the agent in relation to some other object. In certain situations, there may not be a clearly identifiable set of domain specific occurrences with explicitly known occurrence criteria or effects that are definable in terms of a typology of spatial change. However, even in such situations, an analysis of the domain independent events (e.g., event-based evolution of a process) may lead to an understanding of spatio-temporal relationships and help with hypothesis generation [Beller 1991].

3 Modelling Qualitative Spatial Calculi in the Situation Calculus

A study of qualitative spatial calculi from the viewpoint of their formal algebraic properties (e.g., [Ligozat and Renz 2004]) is not relevant in this work. Only the high-level aspects of axiomatic spatial calculi pertaining to different aspects of space such as topology (e.g., region connection calculus [Randell et al. 1992]), orientation (e.g., line-segment based dipole calculus [Moratz et al. 2000], point-based double-cross calculus [Freksa 1992]) that are ubiquitous within the qualitative spatial reasoing domain are of significance in this work. Ontological distinctions pertaining to the nature of primitive spatial entities (regions, points or line-segments) notwithstanding, these spatial calculi are based on similar axiomatic semantics – precisely, these consist of a finite set of jointly exhaustive and pair-wise disjoint (JEPD) relations (1a-1b), compositional inference and consistency maintenance (1c) and the representation of change on the basis of the continuity of the underlying relation space, i.e., based on the conceptual neighbourhood principle [Freksa 1991]. According to this principle, relations r and r' from the relational space are conceptual neighbours if two objects related by r can directly transition to the state of being r' and vice-versa. Depending on the dynamic physical properties (e.g., rigidity, non-rigidity) of the objects involved (see Section 3.3), we assume that changing spatial relationships between objects are the result of motion, continuous deformation or both.

$$(\forall r, r').\ [region(r) \wedge region(r') \rightarrow R_1(r, r') \vee R_2(r, r') \vee \ldots R_n(r, r')] \tag{1a}$$

$$(\forall r, r').\ [\neg R_1(r, r') \wedge R_2(r, r')] \tag{1b}$$

$$R_i(r_1, r_2) \wedge R_j(r_2, r_3) \rightarrow R_k(r_1, r_3) \vee \ldots \vee R_n(r_1, r_3) \tag{1c}$$

$$(\forall r, r')\ [part(r, r') \rightarrow size_{<}(r, r')] \tag{2a}$$

$$(\forall r, r', r'')\ [part(r, r') \wedge front(r', r'') \rightarrow front(r, r'')] \tag{2b}$$

Note that although the examples in (1) use binary relationships, the base relationships could be of arbitrary arity, e.g., binary relations denoting topological relationships of the region connection calculus, ternary relations of the point-based double-cross calculus etc. For brevity, we use binary spatial relationships in all our subsequent examples. Finally, when more than one spatial domain (e.g., topology, orientation, size) is being used in a non-integrated manner, we assume that appropriate axioms of interaction between such inter-dependent aspects are explicitly provided. This is because when interdependent spatial domains are used in a non-integrated manner, spatial relationships from one domain entail the other and vice-versa. For instance, topological and size relations are not independent from each other – some topological relations entail size relations and vice-versa [Gerevini and Renz 2002], e.g., in (2a), it may be observed that a containment relationship between two objects implies a size relationship between them. Another form of interdependence, which is compositional in nature, can bee seen in (2b) where topological and intrinsic-orientation relationships between spatially extended objects interact.

3.1 Situation Calculus as a Representational Formalism

Situation calculus as a representational tool for modelling dynamically changing worlds was first elaborated in [McCarthy and Hayes 1969]. Subsequently, the basic formalism has been considerably extended in order to include support for concurrency and an explicit acount of continuous time [Reiter 2001]. However, in all extensions, the basic ontological elements, viz - *actions/events, situations, and fluents*[2] remain the same. Some notation follows before the situation calculus formalism used in this paper is presented:

By Φ, we denote the set of all spatial fluents, i.e., situation dependent spatial properties of the dynamic system being modelled, with $[|\Phi| \geq 1]$. Depending on the spatial domains being covered, there will be one fluent for each type of spatial relationship between the primitive objects of the domain[3]. For instance, assuming spatially extended objects with intrinsic orientation, one instantiation could involve non-integrated usage of topological, orientation and size relationships: $\Phi \equiv \{\phi_{top}, \phi_{ort}, \phi_{size}\}$. Each $\phi_i \in \Phi$ has a finite denotation set $\Gamma_i \equiv \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, which is determined by the qualitative labels of the respective spatial domain that the fluent is representative of, e.g., $\phi_{rcc8}(r_1, r_2) = \gamma$, where $\gamma \in \{dc, ec, po, eq, tpp, ntpp, tpp^{-1}, ntpp^{-1}\}$. Collectively, we refer to the set of all qualitative labels encompassing all spatial domains being modelled as Γ. Similarly, $\Theta \equiv \{\theta_1, \theta_2, \ldots, \theta_n\}$ denotes the set of all spatial theory specific primitive *spatial transitions* in the theory. A spatial transition refers to a change of qualitative spatial relationship between the entities in the domain. Note that each $\theta \in \Theta$ takes the form of $tran(\gamma, o_i, o_i)$, read as o_i and o_j transition to the state of being in relation γ. Again, γ is one of the finite qualitative spatial relations that may hold between two objects. Recall from the discussion in Section 2 that in so far as a spatial theory is concerned, the only applicable notion of an occurrence is that of a primitive spatial transition definable in it. In this paper, we restrict ourselves to modelling qualitative spatial calculi in the situation calculus. As such, domain specific distinctions involving internal and external events and actions are not utilised here and the only applicable notion of an occurrence is that of a primitive spatial transition definable within the spatial theory being modelled[4].

We adopt the usual convention that all free variables are universally quantified from the outside and that the scope of all quantifications is limited to the respective sort of the particular variable being quantified. Also, note that letters with integral sub-scripts are regarded as constants. Finally, the situation calculus formalism used in this paper is a first-order sorted language with the following 4 classes of axioms:

1. Spatial theory specific possibility criteria for various *spatial transitions* definable in the spatial theory are specified using the binary predicate $Poss(\theta, s)$, where $\theta \in \Theta$. $Poss(\theta, s)$ denotes that the transition θ possible in situation s.
2. A ternary predicate $Holds(\phi, \gamma, s)$ denoting that fluent ϕ has the value γ in situation s. Note that $\phi \in \Phi$. For clarity, we will use it in the following alternative ways: **(a)**. $[\phi(s) = \gamma]$ or **(b)**. $Holds(\phi, \gamma, s))$, the latter essentially being the reified version. A non-determinate situation is expressed in the following manner:$[\phi(s) = \{\gamma_1 \vee \gamma_2\}] \equiv [Holds(\phi, \gamma_1, s) \vee Holds(\phi, \gamma_2, s)]$
3. The binary function $Result(\theta, s)$, which denotes the unique situation resulting from the happening of occurrence θ in situation s. Here, $\theta \in \Theta$.
4. A ternary $Caused(\phi_i, \gamma, s)$ predicate, where $\phi_i \in \Phi$ and $\gamma \in \Gamma_i$, denoting that the fluent ϕ_i is *caused* to take on the value γ in situation s. The $Caused$ predicate will be used to represent the effects of occurrences in the following two ways – *direct effects*, where occurrences are directly stated to effect named fluents via effect axioms and *indirect effects*, where fluents take on values based on the satisfaction of some situation-specific criteria. As such, the $Caused$ predicate is always a direct (direct effects) or indirect link (indirect effects) between fluents and occurrences.

$$Caused(\phi, \gamma, s) \supset Holds(\phi, \gamma, s) \tag{3a}$$

$$\begin{aligned} Poss(\theta, s) \supset \{\neg(\exists\gamma')\, Caused(\phi, \gamma', Result(\theta, s)) \supset \\ Holds(\phi, \gamma, Result(\theta, s)) \equiv Holds(\phi, \gamma, s)\} \end{aligned} \tag{3b}$$

$$When\, i \neq j,\ [\theta_i(\vec{x}) \neq \theta_j(\vec{y})],\quad \theta \in \Theta \tag{4a}$$

$$[\theta_i(\vec{x}) = \theta_j(\vec{y})] \supset [\vec{x} = \vec{y}] \tag{4b}$$

$$When\, i \neq j,\ [\phi_i(\vec{x}) \neq \phi_j(\vec{y})],\quad \phi \in \Phi \tag{4c}$$

$$[\phi_i(\vec{x}) = \phi_j(\vec{y})] \supset [\vec{x} = \vec{y}] \tag{4d}$$

$$true \neq false \wedge (\forall v)\, [v = true \vee v = false] \tag{4e}$$

$$\begin{aligned} [\gamma_1 \neq \gamma_2 \neq \cdots \neq \gamma_n] \wedge \\ (\forall\, \gamma)\, [\gamma = \gamma_1 \vee \gamma = \gamma_2 \vee \cdots \vee \gamma = \gamma_n] \end{aligned} \tag{4f}$$

[2] Time-varying properties of a dynamic system are referred to as *fluents* [Sandewall 1994].

[3] Some more fluents relevant to modelling dynamic object properties will be introduced in Section 3.3.

[4] See [Bhatt et al. 2006a] for details relevant to domain specific distinctions involving internal and external events and actions.

For the predicate $Caused$, we need (3a) denoting that if a fluent ϕ is $Caused$ to take on the value γ in situation s, then ϕ holds the value γ in s. We also include a generic *frame axiom* (3b) thereby incorporating the principle of inertia or the non-effects of occurrences, i.e., unless *caused* otherwise (either directly or indirectly), a fluent's value will necessarily persist. Finally, (4a-4b) and (4c-4d) denote the unique names axioms for occurrences and fluents respectively whereas (4e-4f) represent the domain closure axioms for fluent values.

Explicit Notion of Causality

An explicit notion of causality in the form of the ternary $Caused(\phi,\ \gamma,\ s)$ predicate, which is the source of non-monotonicity within the formalism, is being employed here. Such a notion, in so far as this work is concerned, is primarily used in the following two ways:

1. **Direct Effects of Occurrences**: The basic use of the $Caused$ predicate is to represent the direct effects of know occurrences, e.g., representing the fact that a certain domain specific event *causes* a region to *split*, *grow* or *shrink*. Note that since we are only concerned with modelling domain-independent spatial dynamics in this paper, we refrain from utilising the predicate for representing direct effects. For details concerning this aspect, refer to [Bhatt et al. 2006a].
2. **Indirect Effects**: State constraints constitute an important representational device in our work. As will be evident in the sections to follow, various aspects of a qualitative calculus can be represented using state constraints. However, (some) state constraints also pose serious problems such as containing indirect effects in them. In the context of the situation calculus, Lin [1995], Lin and Reiter [1994] illustrate the need to distinguish ordinary state constraints from indirect effect yielding ones, the latter being also referred to as *ramification constraints*. This is because when ramification constraints are present, it is possible to infer new effect axioms (or simply effects) from explicitly formulated (direct) effect axioms together with the ramification constraints. Simply speaking, ramification constraints lead to what can be referred to as '*unexplained changes*', which is clearly undesirable. Indirect effects also arise when inter-dependent spatial calculi with their respective compositional constraints are used (e.g., refer to (2) and/or section 3.4).

It suffices to point out for now that all such ramification or indirect effect yielding constraints will be represented using the $Caused$ predicate. This way, by minimising the extensionality of the $Caused$ predicate for every relevant situation, causation axioms determining precisely which fluents undergo a change (either directly or indirectly) as a result of named occurrences can be obtained. We postpone the details to Section 3.4.

The Primacy of Change

In our approach, an implicit notion of time is used; a property deriving from our use of the situation calculus formalism for representing and reasoning about change. This is also consistent with the premise that change is more important than the passage of time. As Shoham and Goyal [1988] elaborate: '*The passage of time is important only because changes are possible with time...the concept of time would become meaningless in a world where no changes were possible*'. Consequently, changing properties/fluents (spatial relationships) between the objects involved in the phenomena being modelled will *hold* over *situations* instead of having an explicit temporal parameter. Furthermore, note that we do not commit to a precise nature of a situation, i.e., a situation as either being an instantaneous snapshot of the world, which was the original formulation by McCarthy and Hayes [1969], or alternatively as formulated by Reiter [2001], as a unique node in the overall branching-tree structure (see Fig. 2) of the space of situations starting with the initial situation S_0[5].

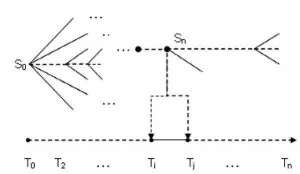

Fig. 2: Tree Structured Situational Space

Various extensions have been provided, most notably by Pinto [1994] and Pinto and Reiter [1995], so as to explicitly accommodate continuous time within situation calculus. The ontological extensions in [Pinto and Reiter 1995] for the representation of time and events are particularly interesting – In their formalism, Pinto and Reiter define a time line (See Fig. 2), which is isomorphic to the non-negative reals, corresponding to a sequence of situations. This sequence essentially corresponds to one directed path (an actual as opposed to a hypothetical evolution), starting at the initial situation, in the overall branching tree structure of situations. If time needs to be accounted for explicitly, the relevant extensions, where every situation corresponds to a time-interval, can be incorporated within the theory by committing to such a branching tree structure of the situations; however, these issues/extensions are beyond the scope of our present aims and will not be addressed here.

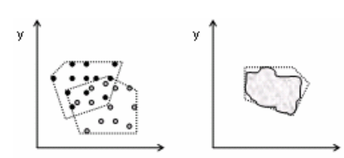

Fig. 3: Convex Hull for Aggregate and Primitive Objects

3.2 A Region Based Spatial Abstraction

In view of the intended applications areas (e.g., cognitive robotics, event-based GIS; see Section 4), we operate within a purely region based framework. However, note that the approach is equally applicable with point or line-segment based

[5] As pointed out by Shanahan [1997], both viewpoints are compatible with the original proposal by McCarthy and Hayes [1969].

calculi. The typical ontological distinction between an object and the region of space it occupies will be made throughout, with the object's *spatial extension* being denoted by the transfer function $space(object)$. For clarity, we sometimes refer to spatial relationships as directly holding between *objects* of the domain instead of their spatial extensions. Whenever necessary, the transfer function can be used to make the necessary distinctions. In order to preserve the generality of the theory for fine-scale analysis with primitive entities or macro-level analysis with aggregates or clusters of entities, we will make some assumptions relevant to the nature of regions in the theory.

1. Regions in the theory correspond to the spatial extents of *objects*, with an object denoting some primitive entity (e.g., material object) or aggregate entity (some collection of objects) that has a well-defined spatiality. The latter scenario is typical of applications in the GIS area (e.g., spatio-temporal analysis in epidemiology, wildlife biology or the study of diffusion processes in general).
2. The size of a region is equivalent to the size the object, which we assume can be defined using some notion of its n-dimensional measure. For instance, if the object is a measurable set in R^n, its size could be its length ($1D$), area ($2D$) or volume ($3D$).
3. The particular interpretation for a region and the notion of its n-dimensional measure has to be *consistent* with regard to the inter-dependent spatial domains being used. For instance, when the available data is qualitatively mapped into the theory, the spatial interpretation for a region, the topological relationships between regions and their corresponding relative sizes (n-dimensional measures) should be consistent with each other.
4. Finally, we assume that the spatial extensions of objects are regular (uniform dimensionality) convex regions of space that approximate the object in question, e.g., using a convex hull primitive or a minimal bounding rectangle for primitive objects or a minimal convex polygon for aggregates of objects (see Fig. 3), the precise semantics of the transfer function (i.e., geometrical interpretation/technique being applied) not being relevant.

3.3 Dynamic Object Properties

Objects in the domain may have varying properties at different times. For example, take the case of a container completely filled with water. In this state, the water can still contain some other object (e.g., dropping a small metal ball in the container). Now lets say that in a later situation, the water is frozen and stays that way for eternity. This change, namely water being solidified into ice, is important and must be reflected as a change of property from a fully flexible to a rigid object. As such, we represent such properties as being situation dependent fluents with details of when and in what manner such changes occur being specifiable only in a domain specific manner. In the following, O and S refer to the set of domain-objects and situations respectively.

- $allows\_containment \subseteq [O \times S]$ – Propositional fluent[6] denoting that a given object may contain other objects.
- $can\_deform \subseteq [O \times S]$ – Propositional fluent denoting that a given object may continuously deform by way of *growth, shrinkage* or change of *shape*.

Another issue is that of classification of objects into rigid and non-rigid types. Consider the following scenarios: (1) A delivery object (o) is lying *disconnected* (dc) next to a delivery vehicle (v) in one situation (s_1) and in a later situation (s_2), is $inside$ the delivery vehicle. Topologically, this is equivalent to the following: $Holds(\phi_{rcc}(o,\ v),\ dc,\ s_1)$ and $Holds(\phi_{rcc}(o,\ v),\ tpp,\ s_2)$. (2) Consider the representation of a bouncing *ball* inside a *room* using purely topological primitives. Here, the state continuously oscillates between tpp and $ntpp$ until eventually steadying at tpp. Since we are dealing with material (rigid) objects[7], this change can be understood to be the result of motion rather than other possibilities such as continuous deformation that are possible with non-rigid objects. However, such a coarse distinction into strictly rigid & non-rigid objects is not sufficient. For example, consider the delivery vehicle (or the room) in the examples aforementioned. Although the object identifying the vehicle cannot *grow* or *shrink*, it can certainly contain other objects. Therefore, the vehicle can neither be classified as being strictly rigid (being in a similar class as that of a metal ball), thereby not allowing interpenetration, nor is it a fully flexible non-rigid object like a water body that can *grow, shrink* or change *shape*. Another interesting issue pertains to the dimensionality of fully-flexible objects such as water or fluids in general – such objects assume the dimensionality of the containing object. An elaborate characterisation of the ontological issues pertaining to the nature of objects is not central to our work. As such, we will not attempt detailed classification of object categories and the kind of changes permissible therein. Our approach will be to make as many distinctions as necessary in the context of our intended application scenarios, e.g., does a particular object *grow* or *shrink*? and whether or not it can *contain* other objects in it?, and specify them in the form of constraints. For instance, assuming that topological relations are involved, the constraint in (5) denotes that if an object cannot *contain* other objects, no object can *transition* (denoted using $trans(...)$) to a state of being a *part* of that object.

$$(\forall\, o,\ o^{'})(\forall s)\ [\neg\ allows\_containment(o,\ s) \supset \neg\ Poss(tran(part,\ o^{'},\ o),\ s)] \tag{5}$$

Note that this approach is very general and issues such as whether the object in question is actually capable of containing some other object (i.e., is its *size* and *shape* suited to contain the target object?) have been ignored. To emphasize, our approach here is to rely on minimal notions of space and

[6] Fluents having a boolean denotation are termed propositional, whereas those having an arbitrary denotation are termed functional.

[7] The vehicle and room can be conceived as a one hollow object bounded by the sides with an opening at one end so as to allow containment relationships with other objects

develop a causal approach for modelling spatial dynamics. When necessary, further distinctions relevant to dynamic object properties can be made by integrating information relevant to other aspects of space[8].

$$rigid(o,\ s)\ \equiv\ [\neg\ allows\_containment(o,\ s) \wedge \neg\ can\_deform(o,\ s)] \tag{6a}$$

$$non\_rigid(o,\ s)\ \equiv\ [allows\_containment(o,\ s) \wedge can\_deform(o,\ s)] \tag{6b}$$

$$\begin{aligned}&Constraints\ on\ rigid\ objects\\&(\forall o,\ o^{'})(\forall s)\ [rigid(o,\ s)\ \wedge\ rigid(o^{'},\ s)\ \supset\\&\quad Holds(\phi_{rcc8}(o,\ o^{'}),\ \gamma,\ s)]\\&\quad where\ \gamma\ \in\ \{dc,\ ec\}\end{aligned} \tag{6c}$$

$$\begin{aligned}&Fully\ flexible\ non-rigid\ Objects:\ General\ case\\&(\forall o,\ o^{'})(\forall s)\ [non\_rigid(o,\ s)\ \wedge\ non\_rigid(o^{'},\ s)\ \supset\\&\quad Holds(\phi_{rcc8}(o,\ o^{'}),\ \gamma,\ s)]\\&where\ \gamma\ \in\ \{dc,\ ec,\ po,\ eq,\ tpp,\ ntpp,\ tpp^{-1},\ ntpp^{-1}\}\end{aligned} \tag{6d}$$

$$\begin{aligned}&Combination\ rigid\ and\ non-rigid\ objects\\&(\forall o,\ o^{'})(\forall s)\ [rigid(o,\ s)\ \wedge\ non\_rigid(o^{'},\ s)\ \supset\\&\quad Holds(\phi_{rcc8}(o,\ o^{'}),\ \gamma,\ s)]\\&where\ \gamma\ \in\ \{dc,\ ec,\ po,\ eq,\ tpp,\ ntpp\}\end{aligned} \tag{6e}$$

$$\begin{aligned}&Semi-rigid\ and\ rigid\ objects\\&(\forall o,\ o^{'})(\forall s)\ [allows\_containment(o^{'},\ s) \wedge\\&\quad \neg can\_deform(o^{'},\ s)\ \wedge\ rigid(o,\ s) \supset\\&\quad Holds(\phi_{rcc8}(o,\ o^{'}),\ \gamma,\ s)]\\&where\ \gamma\ \in\ \{dc,\ ec,\ po,\ eq,\ tpp,\ ntpp\}\end{aligned} \tag{6f}$$

For instance, assuming that a mereo-topological spatial theory is being utilised, a constraint that two material objects can only be either *disconnected* or *externally connected* can be included; see the properties and constraints in (6) that can be used to rule out certain spatial configurations that should not be permitted. Similar properties can be identified for other spatial domains such as orientation, e.g., two objects can only be *connected* from their respective *left* sides or when one object enters (i.e., containment) another one, the intermediate *external connection* and *partial overlap* can only happen via the latter's *intrinsic front*. Note that the constraints such as these or the ones in (6a-6f) essentially form a part of the spatial theory and exist independently of the domain being modelled. This is important in order to enforce a clear separation between a domain independent spatial theory and a domain specific axiomatisation that utilises the general theory. The only requirement here is that that object specific assertions relevant to their dynamic properties will need to be provided by domain-modellers for the scenario under consideration.

[8] In this context, see the classification in [Galton 1993], where Galton identifies categories of permissible transitions between objects by taking into consideration information relevant to other aspects of space.

3.4 General Aspects of Spatial Calculi

Composition Theorems as Ramification Constraints

A straight-forward way to represent every composition theorem is to model it as an ordinary state constraint (7a). However, as discussed previously in Section 3.1, modelling composition theorems in this manner leads to unexplained changes since the resulting constraints contain indirect effects in them. For instance, this is evident whilst performing compositional inference with the spatial relationships involving the trivial case of 3 objects $[o_1, o_2, o_3]$ – if $R_1(o_1,\ o_2)\ \wedge\ R_2(o_2,\ o_3)$, then this constrains the relationship that may hold between o_1 and o_3. As such, from a causal perspective, when either two of the three objects (lets say o_1 and o_2) undergo a transition to a different qualitative state (i.e., $trans(R_i,\ o_1,\ o_2)$), this also has an effect on the relationship between the other two (in this case, o_1 and o_3) since the latter is constrained by the compositional constraints of the relational space. Whilst the details not being relevant here[9], we will apply an explicit notion of causality by utilizing the $Caused(\phi,\ \gamma,\ s)$ predicate for the specification of such ramification constraints (7b). Using this scheme, we will need $8\ \times\ 8$ constraints of the form in (7b).

$$(\forall s)\ [Holds(\phi(o_1,\ o_2),\ \gamma_1,\ s)\ \wedge\ Holds(\phi(o_2,\ o_3),\ \gamma_2,\ s) \supset\ Holds(\phi(o_1,\ o_3),\ \gamma_3,\ s)] \tag{7a}$$

$$(\forall s).\ [Holds(\phi_{rcc8}(o_1,\ o_2),\ \gamma_1,\ s)\ \wedge\ Holds(\phi_{rcc8}(o_2,\ o_3),\ \gamma_2,\ s) \supset\ Caused(\phi_{rcc8}(o_1,\ o_3),\ \gamma_3,\ s)] \tag{7b}$$

Continuity Constraints of Relation Space

In the context of a qualitative theory of spatial change, the most primitive means of change is a explicit change of spatial relationship between two objects (their spatial extensions). To re-iterate, let $tran(\gamma,\ o_i,\ o_j)$ denote such a change, read as, o_i and o_j *transition* to a state of being γ. The possibility axiom for such a transition has been formally expressed in a general manner in (8).

$$\begin{aligned}&Poss(tran(\gamma,\ o_i,\ o_j),\ s)\ \equiv\ [\{space(o_i,\ s)\ =\ r_i \wedge\\&\quad space(o_j,\ s)\ =\ r_j\}\ \wedge\ \{(\exists\ \gamma^{'})\ Holds(\phi(\ r_i,\ r_j),\ \gamma^{'},\ s) \wedge\\&\quad neighbour(\gamma,\ \gamma^{'})\}]\end{aligned} \tag{8}$$

The binary predicate $neighbour(\gamma,\ \gamma^{'})$ in (8) is used to express the possibility of a direct continuous transition (deformation or motion) being consistent between two topological relations and is based on the conceptual neighbourhood principle [Freksa 1991]. According to this principle, relations γ and $\gamma^{'}$ are conceptual neighbours if two objects related by γ can directly transition to the state of being $\gamma^{'}$ and vice-versa. The conceptual neighbourhood graph for a particular set of n spatial relations can be used to define a total of n axioms of the form in (8) so as to comprehensively represent the possibility criteria for every definable spatial transition.

[9] See [Bhatt et al. 2006b] for a more detailed example in the context of RCC-8 topological relations.

Axioms of Interaction between Interdependent Calculi
Axioms of interaction are only applicable when more than one spatial domain is being modelled in an non-integrated manner. They refer to an explicit characterisation of the relative entailments that exist between inter-dependent aspects of space. Note that the entailments may be non-determinate; however, they will still need to be explicitly axiomatised as ramification constraints (7b). For instance, size *equality* rules out all containment (tpp, $ntpp$ and their inverses) relationships. Similarly, if it is known that object o is a tangential part of object $o^{'}$, then it is implicitly known that the size of object o is less than the size of $o^{'}$ (see (9)).

$$(\forall o, o^{'})(\forall s).\,[space(o,\, s) \,=\, r \,\wedge\, space(o^{'},\, s) \,=\, r^{'} \,\wedge \\ Holds(\phi_{rcc8}(r,\, r^{'}),\, tpp,\, s) \,\supset \\ Caused(\phi_{size}(r,\, r^{'}),\, <,\, s)] \tag{9}$$

Causal Laws of the Spatial Theory
Successor state axioms (SSA) specify the causal laws of the spatial theory being modelled, i.e., what changes as a result of various occurrences in the system being modelled. Generally, the SSA is based on a *completeness assumption* which essentially means that all possible ways in which the set of fluents may change is explicitly formulated, i.e., there are no indirect effects [Reiter 1991]; we refer to this SSA as the Pseudo successor state axiom (PSA). The SSA that needs to be derived here, referred to as SSA-Proper, must also account for indirect effect yielding state constraints – Recall the use of the causal relation $Caused(\phi,\, \gamma,\, s)$ in (7b) toward the representation of the composition table theorems and axioms of interaction in addition to direct effects. What remains to be done is to minimize the causal relation by circumscribing it (or using some other form of minimization) with the following set of axioms fixed – the foundational axioms in (3a-4f), the ramification constraints of the form in (7b) (i.e., compositional constraints and axioms of interaction) and the transition pre-conditions of the form in (8). The result of minimization is the *Causation Axiom* in (10a).

$$Caused(\phi(o_i, o_k),\, \gamma_k,\, s) \,\equiv \\ [\{(\exists o_j,\, \gamma_i,\, \gamma_j)\, Holds(\phi(o_i,\, o_j),\, \gamma_i,\, s) \wedge \\ Holds(\phi(o_j,\, o_k),\, \gamma_j,\, s)\} \vee \\ \{(\exists\, \gamma_l)\, Holds(\phi^{'}(o_i,\, o_k),\, \gamma_l,\, s)\}] \\ where\, \phi,\, \phi^{'} \,\in\, \Phi \tag{10a}$$

$$Poss(\theta,\, s) \,\supset\, [Holds(\phi(o_i,\, o_j),\, \gamma_i,\, Result(\theta,\, s)) \,\equiv \\ \{(\forall\, \gamma^{'})\, Holds(\phi(o_i,\, o_j),\, \gamma_i,\, s) \,\wedge\, \theta \,\neq\, tran(\gamma^{'},\, o_i,\, o_j)\} \vee \\ \{\theta \,=\, tran(\gamma_i,\, o_i,\, o_j)\}] \tag{10b}$$

$$Poss(\theta,\, s) \,\supset\, [Holds(\phi(o_i,\, o_j),\, \gamma_i,\, Result(\theta,\, s)) \,\equiv \\ \{(\forall\, \gamma^{'})\, Holds(\phi(o_i,\, o_j),\, \gamma_i,\, s) \,\wedge\, \theta \,\neq\, tran(\gamma^{'},\, o_i,\, o_j)\} \vee \\ \{\theta \,=\, tran(\gamma_i,\, o_i,\, o_j)\} \vee \\ \{Caused(\phi(o_i,\, o_j),\, \gamma_i,\, Result(\theta,\, s))\}] \tag{10c}$$

$$Poss(\theta,\, s) \,\supset\, [Holds(\phi(o_i,\, o_j),\, \gamma_i,\, Result(\theta,\, s)) \,\equiv \\ \{(\forall\, \gamma^{'})\, Holds(\phi(o_i,\, o_j),\, \gamma_i,\, s) \,\wedge\, \theta \,\neq\, tran(\gamma^{'},\, o_i,\, o_j)\} \vee \\ \{\theta \,=\, tran(\gamma_i,\, o_i,\, o_j)\} \vee \\ \{(\exists o_k, \gamma_j,\, \gamma_k)\, Holds(\phi(o_i,\, o_k),\, \gamma_j,\, s) \wedge \\ Holds(\phi(o_k,\, o_j),\, \gamma_k,\, s)\} \vee \\ \{(\exists\, \gamma_l)\, Holds(\phi^{'}(o_i,\, o_j),\, \gamma_l,\, s)\}] \;\; where\, \phi,\, \phi^{'} \,\in\, \Phi \tag{10d}$$

The causation axiom (10a) must be integrated with a Pseudo-SSA (PSA) (10b)(PSA is SSA without indirect effects) to derive the SSA-Proper in (10c). More appropriately, the final result is the SSA-Proper in (10d). To re-iterate, the effect of minimising the causal relation is to derive the causation axioms which essentially includes contextual conditions (direct or indirect) that could possibly *cause* a fluents value to change. This causation axioms is then compiled with the PSA in order to obtain the SSA-Proper. A step-by-step illustration of this approach (in a general context) that utilises circumscription as the minimisation technique can be found in [Lin 1995, Lin and Reiter 1994].

JEPD and Other Properties
The property of the base spatial relationships being jointly exhaustive and mutually disjoint can be expressed using ordinary state constraints of the form such as in (7a) in a straightforward manner. In general, we need a total of n state constraints of the form in (11a) to express the jointly-exhaustive property of a set of n base relations.

$$(\forall s).\, \neg[Holds(\phi(o_1,\, o_2),\, \gamma_1,\, s) \,\vee\, Holds(\phi(o_1,\, o_2),\, \gamma_2,\, s) \\ \vee \cdots \vee\, Holds(\phi(o_1,\, o_2),\, \gamma_{n-1},\, s)] \,\supset \\ Holds(\phi(o_1,\, o_2),\, \gamma_n,\, s) \tag{11a}$$

$$(\forall s).\, \neg\, [Holds(\phi(o_1,\, o_2),\, \gamma_1,\, s) \,\wedge\, Holds(\phi(o_1,\, o_2),\, \gamma_2,\, s)] \tag{11b}$$

$$(\forall s).\, [Holds(\phi(o_i,\, o_j),\, \gamma,\, s) \,\supset \\ Holds(\phi(o_j,\, o_i),\, \gamma,\, s)] \tag{12a}$$

$$(\forall s).\, [Holds(\phi(o_i,\, o_j),\, \gamma,\, s) \,\supset \\ \neg\, Holds(\phi(o_j,\, o_i),\, \gamma,\, s)] \tag{12b}$$

Similarly, $[n(n \,-\, 1)/2)]$ constraints of the form in (11b) are sufficient to express the pair-wise disjointness of n relations. Additionally, other miscellaneous properties such as the symmetry (12a) & asymmetry (12b) of the base relations too can be expressed using ordinary state constraints.

3.5 Initial State of the World - Big Bang Situation

A description of initial fluent values when no occurrences have happened is needed: For spatial fluents, there exist 2 classes: those which model the spatial relationship between objects (e.g., topological or orientation relationships) & those which characterise the dynamic object properties (e.g., $allows\_containment$). The case for dynamic object properties is trivial and will be excluded whereas that of non-spatial fluents (i.e., domain specific dynamic properties) is not applicable in the context of the spatial theory.

$$\Omega \equiv [Holds(\phi_{rcc8}(o_1,\ o_2),\ tpp,\ S_{init}) \wedge Holds(\phi_{rcc8}(o_2,\ o_3),\ dc,\ S_{init}) \wedge Holds(\phi_{size}(o_2,\ o_3),\ =,\ S_{init})] \tag{13a}$$

$$\Omega \wedge \Sigma_{CT} \wedge \Sigma_{INT} \vdash \Omega^{'}$$

$$where\ \Omega^{'} \equiv [Holds(\phi_{rcc8}(o_1,\ o_2),\ tpp,\ S_{init}) \wedge Holds(\phi_{rcc8}(o_2,\ o_3),\ dc,\ S_{init}) \wedge Holds(\phi_{rcc8}(o_1,\ o_3),\ dc,\ S_{init}) \wedge Holds(\phi_{size}(o_1,\ o_2),\ <,\ S_{init}) \wedge Holds(\phi_{size}(o_2,\ o_3),\ =,\ S_{init}) \wedge Holds(\phi_{size}(o_1,\ o_3),\ <,\ S_{init})] \tag{13b}$$

As for the fluents encompassing spatial relationships, the *initial situation* (S_{init}) description involving n domain objects requires a complete $n-clique$ specification with $[n(n-1)/2]$ spatial relationships of one type (spatial domain); this can either be supplied explicitly or can be derived from a partial specification. When relationships between some objects are omitted, a complete description of S_{init} (with disjunctive labels) can be derived on the basis of the composition theorems for the spatial domain under consideration. As an example, consider the simplest case involving topological and size relationships between 3 objects in (13): Ω denotes a partial description involving the 3 objects, viz - the relationship between objects o_1 and o_3 is unknown. Given Ω, $\Omega^{'}$ can be monotonically derived ($\vdash$) on the basis of Ω and the RCC-8 composition theorems (Σ_{CT}) and axioms of interaction between topology and size (Σ_{INT}). Here, $\Omega^{'}$ is a monotonic extension of Ω in the sense that whilst new information is conjoined with Ω, none of the existing spatial knowledge is invalidated.

4 Applications

From the proposed causal framework and the structure and semantics of the situation calculus, computational tasks such as explanation, planning and projection directly follow. Within the specialised spatial reasoning domain, these translate to causal explanation (i.e., inferring cause from observations), spatial planning/re-configuration and spatial simulation. We are primarily interested in causal explanation and spatial planning in the context of dynamic GIS and cognitive robotics respectively. In the following sections, a brief discussion of the said applications follow.

Causal Explanation of Dynamic GeoSpatial Phenomena

Causal explanation is the process of retrospective analysis by the extraction of an event-based explanatory model from available spatial data (e.g., temporally-ordered snap-shots). Indeed, the explanation is essentially an event-based history of the observed spatial phenomena defined in terms of both domain-independent and domain-dependent occurrences. Causal and, if applicable, telic accounts of a process being modelled are applicable in a diverse range of geospatial phenomena, such as movement of clusters of animals (wildlife biology), monitoring people-clusters in times of crisis on the basis of GPS-based positional information (e.g., emergency and disaster management and planning, defence modelling and simulation) and even in the geospatial analysis of the spread of diseases (epidemiology), where an event-based model can be extracted (or evolution of the phenomena be defined) on the basis of the typology of fundamental spatial changes. Additionally, causal analysis is also applicable in real-time surveillance systems where the occurrence criteria for domain-specific events/actions can be defined on the basis of certain, possibly incompletely known, spatial-configurations of the domain objects and/or the patterns of their dynamic evolution. Using this approach it is possible to explain spatial phenomena at a higher-level either in terms of domain-specific occurrences that *cause* the observed changes or alternatively, in a domain-independent manner on the basis of a fundamental typology of spatial change such as *splitting*, *growth*, *movement* etc. Along these lines, we are investigating issues relevant to modelling of the dynamic spatio/temporal evolution of aggregate or cluster-oriented phenomena for a public-health domain case-study (see [Bhatt and Whigham 2006] for initial position description).

A Goal-Directed Control Mechanism for Spatial Planning

Given the background spatial theory (i.e., one or more spatial domains modelled using the causal approach), domain specific constraints, an initial state and an overall objective to be achieved, one task is to derive a sequence of (spatial) actions that will fulfil the desired objective. In other words, how do we transform one spatial configuration into another? Or alternately, what are the spatial transformations that are necessary corresponding to the achievement of a certain goal? Here, a goal can be a situation in which a certain action has happened or where some fluents hold specific values. Note that this problem, which can be considered akin to the task of arriving at a desired spatial configuration starting at a initial configuration, is one of the simplest form of spatial planning. Variations along this line involve the incorporation of dynamically available information (e.g., sensing-abilities of a robot) in the planning process, since an incremental plan generation approach, where sensing affects subsequent planning, is more powerful in comparison to an off-line or static approach.

5 Discussion and Outlook

By regarding spatial theories as a specialisation within a higher-level framework to reason about change in general, spatial theories can be directly utilised in application domains involving reasoning about dynamic spatial phenomena (e.g., cognitive robotics, event-based GIS). With this objective, we have presented the basic outline of a causal theory involving explicit reasoning about events, actions and their effects for representing and reasoning about spatial dynamics. A step-by-step illustration of the manner in which different aspects of axiomatic qualitative spatial calculi may be accounted for within such a causal framework is presented.

Several extensions to the basic theory are possible: (1) Presently, it is not possible to represent the (abrupt or explicitly known) *appearance/disappearance* of a spatial object. Refinements are needed at the foundational level in order to allow this behaviour. One approach is to maintain facts about the existence of regions (at the foundational level) and propagate them backwards in the situation-based history whenever appearance/disappearance events occur[10]. (2) An important extension is to exploit the non-monotonic reasoning capability within the formalism for the representation of human-like common-sense reasoning with incomplete information. Although non-monotonicity is presently used within the formalism, that is done for entirely different reasons (see section 3.1). (3) Concurrency is an issue that has not been investigated in the specialised spatial reasoning domain. However, several extensions to the basic situation calculus formalism in the form of high-level programming languages (e.g., conGolog [Giacomo et al. 2000], ccGolog [Grosskreutz and Lakemeyer 2000]) exist for representing and reasoning about concurrent phenomena in general. Since the representation of qualitative theories of space/spatial dynamics into situation calculus is achieved, how features of the extended situation calculus formalism (e.g., concurrency, explicit time) might be usable for the representation of concurrency in the spatial domain is an important next step.

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[10] This line-of-thought draws on the work in [Gooday and Cohn 1996], where Gooday and Cohn solve the same problem in the context of the transition calculus using a forward and backward completion mechanism.

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