# Qualitative Spatial Scene Modeling for Ambient Intelligence Environments

Frank Dylla and Mehul Bhatt

SFB/TR 8 Spatial Cognition, Universität Bremen, Germany {dylla,bhatt}@sfbtr8.uni-bremen.de

**Abstract.** In ambient intelligence systems, it is necessary to represent and reason about dynamic spatial scenes and configurations. Primarily, the ability to perform predictive and explanatory analyses on the basis of available sensory data is crucial toward serving a useful intelligent function within such environments. In this paper, we present a qualitative model for representing the relevant aspects of these environments in an adequate manner. The model is suited for reasoning about spatial configurations and dynamics in spatial environments. We clarify and elaborate on our ideas with examples grounded in a smart home environment.

### 1 Introduction

A wide-range of application domains in Artificial Intelligence, from cognitive robotics to intelligent systems encompassing diverse paradigms such as ambient intelligence and ubiquitous computing environments, require the ability to represent and reason about spatial scenes or configurations and how they might evolve over time. For instance, real world ambient intelligence systems that monitor and interact with an environment populated by humans and other artefacts require a formal means for representing and reasoning with spatio-temporal and event-based phenomena that are grounded in the environment being modeled. Here, the location of a mobile-object, e.g., a person or animal, may require to be projected within the spatial environment at hand, e.g., smart homes, airports, or traffic junctions, for the purpose of dynamic scene analyses and interpretation, event-recognition, alert generation, and so forth. Similarly, the unfolding of sequences of spatial configurations that correspond to certain activities within the application domain of interest may be required to be modeled too, e.g., in the form of causal explanation of observations on the basis of the actions and events that may have caused the observed state-of-affairs. A fundamental requirement within such application domains is the representation of dynamic knowledge pertaining to the spatial aspects of the environment within which an agent (e.g., a robot) or a system is functional, e.g., a monitoring or alert generation system in a smart-home [1,2]. Furthermore, it is also desired that the perceivable variations in space be explicitly linked with the functional aspects of the environment being modeled and reasoned about - in other words, it is necessary to explicitly take into consideration the fact that perceivable changes in the surrounding space

C. Xiong et al. (Eds.): ICIRA 2008, Part I, LNAI 5314, pp. 716–725, 2008.

<sup>©</sup> Springer-Verlag Berlin Heidelberg 2008

are typically the result of interaction (i.e., events, actions) within the environment. Therefore, a qualitative representation of the environmental space and its potential changes is necessitated.

In this paper, we propose to utilize a formal basis for representing and reasoning about space, within ambient intelligence systems or in general, within a ubiquitous computing environment. The model utilizes formal spatial calculi [3] for representing and reasoning about space in a qualitative manner within the aforementioned domains of interest. Precisely, the model is based on existing qualitative theories of space, or qualitative spatial calculi, pertaining to differing spatial domains, such as topology [4] and orientation [5, e.g.,]. The key advantage of qualitative spatial representations is that qualitative characterizations of space are well-suited for high-level reasoning and decision-making with incomplete information – that is to say, such representations are cognitively adequate. By abstracting objects to geometric primitives such as points, lines and regions and partitioning infinite quantity or metric spaces to finite qualitative categories, qualitative spatial reasoning is able to capture distinctions between objects and the relationships between them that make an important qualitative difference, but ignore others [3]. Given the semantics of the calculus and how relations between objects might change with respect to continuous time and persistent objects (conceptual neighborhood), key reasoning tasks involving projection, planning and explanation directly follow.

In section 2, we present an overview of qualitative methods in spatial representation and reasoning. The domain of 'qualities' as a means for high-level spatial abstraction and reasoning is illustrated and key results, specifically, qualitative spatial calculi, in the area are cited. In Section 3 we illustrate the proposed qualitative model and elaborate on requirements with respect to applying and reasoning with the qualitative formalization. We close with a summary and outlook of the work presented.

### 2 Qualitative Spatial Representation and Reasoning

Qualitative Reasoning (QR) is concerned with capturing everyday commonsense knowledge of the physical world with a limited set of symbols and relationships and manipulating it in a non-numerical manner [3]. The subfield of qualitative reasoning that is concerned with representations of space is called Qualitative Spatial Reasoning (QSR). We introduce the essence of qualitativeness in spatial reasoning and how such representations are applied to formulate constraints on spatial configurations within a dynamic spatial environment.

#### 2.1 Qualitative Spatial Calculi

The main aim of research in qualitative methods in spatial representation and reasoning is to develop powerful representation formalisms that account for the multi-modality of space in a cognitively acceptable way [6,3]. A qualitative spatial description captures distinctions between objects that make an important

qualitative difference but ignores others. In general, objects are abstracted to geometric primitives e.g. points, lines, or regions in the Euclidian plane and discrete systems of symbols, i.e., finite vocabularies, are used to describe the relationships between objects in a specific domain. A complete model for a certain domain is called a *qualitative calculus*. It consists of the set of relations between objects from this domain and the operations defined on these relations. In general, spatial calculi can be classified into two groups: topological and positional calculi [7]. Topological calculi are, for instance, the region-based calculi RCC-5 or RCC-8 [4] or the Cyclic Interval Calculus [8]. Positional calculi, i.e. calculi dealing with orientation or distance information, are for example the Double-Cross Calculus [9] and the Dipole Calculus [10]. In general, two aspects, namely the static and the dynamic aspect, are important from a reasoning viewpoint for any spatial calculus:

**I.** Static Aspect – Reasoning based on Compositions and Constraint Satisfaction: Relations of a spatial calculus are used to formulate constraints about the spatial configuration of objects from the domain of the calculus. This results in the specification of a spatial *constraint satisfaction problem* (CSP) which can be solved with specific reasoning techniques, e.g. by applying composition and intersection operations on the incorporated relations. A prerequisite for applying constraint-based reasoning techniques is a set of *base relations*  $\mathcal{BR}$ , also called primitive relations, which are *jointly exhaustive and pairwise disjoint* (JEPD). In the case of binary relations, JEPD means that for any pair of entities exactly one base relation holds. For arbitrary *n*-ary calculi this must hold for any *n*-tuple. The composition results can be precomputed and stored in so-called *composition tables* (CT). From an axiomatic viewpoint, each entry of such a composition table is in actuality a (composition) theorem of the form:  $(\forall a, b, c)$ .  $R_1(a, b) \land R_2(b, c) \to R_3(a, c)$ . Compositions are further discussed in sections 2.2 and 2.3 where we illustrate these concepts with specific spatial calculi.

**II.** Dynamic Aspect – Conceptual Neighborhood based Reasoning: Spatial neighborhoods are very natural perceptual and cognitive entities [11]. These extend static qualitative representations by interrelating the discrete set of base relations by the temporal aspect of transformation of the basic entities. Two spatial relations of a qualitative spatial calculus are conceptually neighbored, if they can be continuously transformed, by motion and/or continuous deformation, into each other without resulting in a third relation in between. However, the term *continuous* with regard to transformations, such as locomotion, growing or shrinking, or deformation, result in different neighborhood structures. We illustrate the principle of conceptual neighborhoods for topological and orientation calculi in sections 2.2 and 2.3.

# 2.2 Topology and the Region Connection Calculus (RCC)

Topological distinctions are inherently qualitative in nature and they also represent one of the most general and cognitively adequate ways for the representation

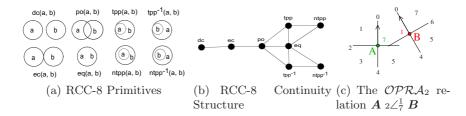


Fig. 1. Topological and orientation primitives

of spatial information [12,13]. The prevalent axiomatic approach to building topological theories of space in the QSR domain has its roots in the philosophical logic community, most notably [14,15]. Following the this work, the class of axiomatic topological theories referred to as *Region Connection Calculus* (RCC) have been developed [4]. The work by Egenhofer and Franzosa adopts a point-set theoretic approach and is based on conventional mathematical topology [16].

The RCC-8 Fragment. RCC is a modification and extension of the Clarke's original region-based theory. The basic part of the formal theory assumes a dyadic relation of *connection*, namely C(a, b), denoting that region a is connected to region b. Topologically, this has the interpretation that the topological closures of a and b share at least one point. From the primitive of *connec*tion, the mereological relation of *parthood* is defined which is, in turn, used to define proper-part (PP), overlap (O) and disjoint (DR). Further, the relations disconnected (DC), externally connected (EC), partial overlap (PO), equal (EQ), tangential proper-part (TPP) and non-tangential proper-part (NTPP) are defined. These relations, along with the inverses of the last two, namely  $\mathsf{TPP}^{-1}$ and  $NTPP^{-1}$  constitute a JEPD set of base relationships, commonly referred to as the RCC-8 fragment of the region connection calculus. Figure 1(a) is a 2D illustration of the topological relationships that constitute the RCC-8 fragment. Reasoning about static spatial descriptions within the RCC framework, or within any spatial calculus relevant to a differing aspect of space that is based on similar semantics, is performed by way of *composition*. For instance, if it is know that aand b are disconnected and that c is tangential part of b, then, by composition, it is also known that a and c are *disconnected*. It is on the basis of such composition theorems that the composition table is constructed or pre-defined in order to exploit it for constraint-based reasoning. Finally, in so far as the dynamics are concerned, the continuity structure formed by the underlying relation space (see Fig. 1(b)) is utilized for modeling changing spatial descriptions.

#### 2.3 Orientation and the $OPRA_m$ Calculus

For representing relative orientation information we apply the  $OPRA_m$  calculus [5]. The calculi in this family are designed for reasoning about relative orientation relations between *oriented points* (points in the plane with an additional

direction parameter) and are well-suited for dealing with objects that have an intrinsic front or move in a particular direction. An oriented point O can be described by its Cartesian coordinates  $x_O, y_O \in \mathbb{R}$  and a direction  $\phi_O \in [0, 2\pi)$  with respect to an absolute frame of reference. With the parameter m the angular resolution can be influenced, i.e. the number of base relations is determined.

In the case of  $\mathcal{OPRA}_2$ , the orientation calculus we apply in our examples, for each pair of oriented points, 2 lines are used to partition the plane into 4 planar and 4 linear regions (see Fig. 1(c)). The orientation of the two points is depicted by the arrows starting at A and B, respectively. The regions are numbered from 0 to 7, where region 0 always coincides with the orientation of the point. An  $\mathcal{OPRA}_2$  base relation is a pair (i, j), where i is the number of the region, seen from A, that contains B and j vice versa. These relations are written as  $A_2 \angle_i^j B$ . Additional base relations describe situations in which both oriented points are at the same position. However, these are not of particular interest in this work. In [17] the general neighborhood structure of  $\mathcal{OPRA}_m$  is derived. Regarding the specific granularity m = 2 and that in our application o-points cannot coincide. The restricted neighborhood structure for the task at hand is given by:

$$\mathtt{cn}_g({}_{2}\!\angle_i^j) = \{{}_{2}\!\angle_{i-1}^{j-1}, {}_{2}\!\angle_{i-1}^{j}, {}_{2}\!\angle_{i-1}^{j+1}, {}_{2}\!\angle_i^{j-1}, {}_{2}\!\angle_{i+1}^{j+1}, {}_{2}\!\angle_{i+1}^{j}, {}_{2}\!\angle_{i+1}^{j+1}, {}_$$

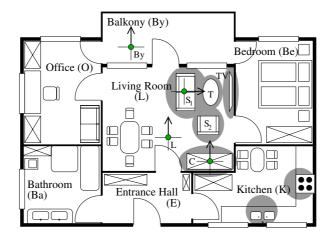
# 3 Qualitative Spatial Scene Descriptions

### 3.1 Scene Description Ontology with Topological and Orientation Primitives

A 'spatial scene description ontology' that is firmly rooted in existing qualitative spatial calculi (section 2.1) and is general enough to be used in varied dynamic spatial scenarios in ambient intelligence systems is essential. Depending on the degree of formalization or richness of the spatial theory being employed, spatial scene descriptions in ambient environment primarily consist of qualitative spatial relationships relevant to one or more spatial dimensions (e.g., topology, orientation, direction, size). Since we need to model containment (e.g., in a room) and also direction of motion (of an agent) or orientation of objects relative to one another, a mixed ontology of regions of space and oriented-points is sufficient for our scene description purposes. Using the example of a smart home environment (cf. Fig. 2), we illustrate the manner in which some typical spatial scenes in such environments may be qualitatively modeled using a basic spatial scenes in such environments may be qualitatively modeled using a basic spatial scene description ontology that is grounded in formal spatial calculi relevant to differing aspects of space, precisely the topological calculus RCC-8 and the relative orientation calculus  $OPRA_m$  from sections 2.2 and 2.3 respectively.

### 3.2 Complete Spatial Scene Descriptions

A 'situation' is a unique node within the overall branching-tree structure (see Fig. 3(c)) of the space of situations starting with the initial situation  $S_0$ . In

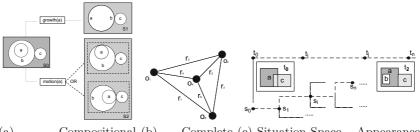


**Fig. 2.** Layout of an application environment (smart home). The axes of o-points (cf. Section 2.3) within rooms or objects denote the intrinsic fronts of these objects, e.g., the living room (L) and the sofa  $(S_1)$ . The grey areas around objects indicate the functional space of the objects, i.e., the area in which interaction may be possible.

$\phi_{top}/\phi_{ort}$	Ву		0		L		Be		E		$S_1$		$S_2$		т		TV		С	
By	×	х	EC	$2 \angle_{1-3}^{5-7}$	EC	$2 \angle_{3-5}^{7-1}$	EC	$2 \angle_{5-7}^{1-3}$	DC	-	DC	-	DC	-	DC	-	DC	-	DC	-
0	EC	$2 \angle_{5-7}^{1-3}$	×	x		$2 \angle {}^{1-3}_{5-7}$	DC		DC	-	DC	-	DC	-	DC	-	DC	-	DC	-
L	EC	$2 \angle {}^{3-5}_{7-1}$	EC	$2 \angle_{1-3}^{5-7}$	х		EC	$2 \angle {1-3 \atop 5-7}$	EC	$2 \angle_{3-5}^{7-1}$	PP	-	PP	-	PP	-	PP	-	PP	-
Be	EC	$2 \angle {5-7 \atop 1-3}$	DC	-	EC	$2 \angle_{1-3}^{5-7}$	×		DC		DC	-	DC	-	DC	-	DC	-	DC	-
E	DC	-	DC	-	EC	$2 \angle_{7-1}^{3-5}$	DC	-	х	x	DC	-	DC	-	DC	-	DC	-	DC	-
$S_1$	DC	-	DC	-	PP	-	DC	-	DC	-	х	х	DC	$2 \angle \frac{1}{7}$	DC	$2 \angle_{0}^{0}$	DC	$2 \angle_{0}^{0}$	DC	$2 \angle \frac{1}{7}$
$S_2$	DC	-	DC	-	PP	-	DC	-	DC	-	DC	$2 \angle_1^7$	х	х	DC	$2 \angle_{0}^{2}$	DC	$2 \angle \frac{1}{7}$	DC	$2 \angle_4^0$
Т	DC	-	DC	-	PP	-	DC	-	DC	-	DC	$2 \angle_{0}^{0}$	DC	$2 \angle_{2}^{0}$	×	х	DC	$2 \angle_4^0$	DC	$2 \angle_{2}^{0}$
TV	DC	-	DC	-	PP	-	DC	-	DC	-	DC	$2 \angle_{0}^{0}$	DC	$2 \angle_1^7$	DC	$2 \angle_0^4$	х	х	DC	$2 \angle_1^7$
С	DC	-	DC	-	PP	-	DC	-	DC	-	DC	$2 \angle_1^7$	DC	$2 \angle_0^4$	DC	$2 \angle_{0}^{2}$	DC	$2 \angle \frac{1}{7}$	х	х

Table 1. An exemplary part of the scene description matrix

other words, a situation includes a complete history of the evolution of the system in terms of the primitive occurrences that have occurred starting in the initial situation  $S_0$ . Corresponding to each such situation, there exists a situation description that characterizes the 'state' (henceforth 'situation state') of the system. Note that although a certain state may re-occur during the evolution of the system, each such re-occurrence corresponds to a unique 'situation' within the overall branching-tree structure of the situational space, because although the states are equal, the history of actions and occurences that led to each of the states are different. Starting with the initial situation, it is necessary that the spatial component of every situation state be a complete specification without any missing information. Note that by 'complete specification', we do not imply absence of uncertainty or ambiguity. Completeness also includes those instances where the uncertainty is expressed as a set of completely specified alternatives



(a) Compositional (b) Complete (c) Situation Space – Appearance – Constraints and Ramifi- N-Clique De- Explanation cations scription

Fig. 3. Spatial Scene Descriptions

in the form of disjunctive information. The initial situation basically includes a specification of initial fluent values. The fluents can be broadly categorized in spatial fluents and aspatial fluents. The category of spatial fluents consist of domain-specific dynamic properties that are not spatial in nature. For spatial fluents, at least from the viewpoint of this paper, there exist *two* broad categories:

(I). Existential Facts: These are propositional fluents that provide an explicit existential characterization of every spatial object that is known to exist in the initial situation. It is necessary for every known object to *exist* in the initial situation. As such, for a domain consisting of n objects in the initial situation, n existential facts to this effect are required.

(II). Spatial Relationships: These model the spatial relationships that exist between the objects that are known to exist in the initial situation. For every type of spatial relationship being modeled, the *initial situation* description involving n domain objects requires a complete n-clique specification (see Fig. 3(b)) with [n(n - 1)/2] spatial relationships of one type or spatial domain. Albeit with a certain level of uncertainty, this may either be specified explicitly or can be derived (e.g., by composition and mutual entailments between spatial domains) from the explicitly provided partial specification.

The 'scene description matrix' in Table 1 illustrates the concept of a complete spatial description for the smart home environment depicted in Fig. 2. For lack of space, the spatial relationships of some objects have been omitted. Observe that the information contained in Table 1 is grounded in the spatial vocabulary of the RCC-8 and the oriented point relation algebra  $\mathcal{OPRA}_m$  from section 2.1. Indeed, the initial situation description thus obtained (from the above discussed categories of properties) concerns requirements as stipulated by a domain-independent spatial theory. Further additions by way of domainspecific properties, event and action specifications and other aspects in conformity with the requirements of a complete domain theory are essential in order to execute reasoning tasks within the propsed formalization.

#### 3.3 Global Consistency of Spatial Information

Spatial situation descriptions denoting configurations of domain objects, i.e., by way qualitativ spatial relationships relevant to one or more spatial dimensions that hold between the objects of the domain, must be globally consistent in adherence to the compositional constraints of the underlying qualitative space. The notion of compositional consistency also includes those scenarios when more than one aspect of space is being modeled in a non-integrated way, i.e., relative dependencies between mutually dependent spatial dimensions should be taken modeled explicitly. Ensuring these two aspects of global consistency of spatial information is non-trivial because the compositional constraints contain indirect effects in them thereby necessitating a solution this problem. In the field of reasoning about action and change the problem of indirect effects<sup>1</sup> is called the ramification problem. In the spatial representation task, i.e., the embedding of qualitative spatial calculi within a spatial theory, indirect effect yielding constraints are a recurring problem – modeling composition theorems and axioms of interaction may lead to unexplained changes since the resulting constraints contain indirect effects in them [19]. For instance, consider the illustration in Fig. 3(a) – the scenario depicted herein consists of the topological relationships between three objects 'a', 'b' and 'c'. In the initial situation ' $S_0$ ', the spatial extension of 'a' is a non-tangential part of that of 'b'. Further, assume that there is a change in the relationship between 'a' and 'b', as depicted in Fig. 3(a), as a result of a direct effect of an event such as growth or an action involving the motion of 'a'. Indeed, as is clear from Fig. 3(a), for the spatial situation description in the resulting situation (either  $S_1$  or  $S_2$ ), the compositional dependencies between 'a', 'b' and 'c' must be adhered to, i.e., the change of relationship between 'a' and 'c' must be derivable as an indirect effect. In a trivial scenario, such as the present one, consisting of few objects, it could be correctly argued that the indirect effects can be completely formulated as direct effects. However, for a more involved scene description n objects and complete *n*-clique descriptions consisting of n(n - 1)/2 spatial relationships for every spatial domain (e.g., topology, orientation, size) being modeled is impractical and error prone. The situation is only complicated given that fact that some of the spatial domains being modeled could be inter-dependent.

Whilst the details not being relevant here, it suffices to point out that a solution to the problem of ramifications for this particular case is obtainable from the general works in [20,18]. The solution basically involves appeal to causality and non-monotonic reasoning to minimize the effects of occurrences whilst deriving the causal laws of the domain.

### 4 Summary and Outlook

We have shown how spatial environment and task modeling in ambient intelligence systems can be achieved adequately by means of qualitative spatial

 $<sup>^{1}</sup>$  The general problem indirect-effect yielding state constraints is elaborated on in [18].

reasoning. The relations of qualitative spatial calculi serve as a basis for the qualitative world model. In the example of a smart home environment, the proposed model contains topological knowledge (RCC-8) and relative orientation knowledge ( $\mathcal{OPRA}_m$ ) about the objects. The spatial dynamics of the model are given by the conceptual neighborhood structures of the applied calculi. For the integration of the two approaches we addressed the connections between global consistency in qualitative spatial reasoning and the ramification problem in reasoning about action and change.

In the next steps based on the considerations presented here we derive a spatial theory based on RCC-8 and  $OPRA_m$  with additional domain dependant motion patterns that potentially characterize activities. We will represent the proposed spatial model and according activities in terms of the situation calculus. With such a spatial theory we will be able to draw conclusions and causal explanations in the spatial formalization at hand. Additionally, we will investigate how qualitative methods can be combined with quantitative methods to develop distinct measures for scene analysis purposes, e.g. for deriving different spatial and aspatial distance measures between configurations, or for estimating the similarities between different shapes. Another future line of work is the construction of domain independent motion patterns.

### References

- Gottfried, B., Guesgen, H.W., Hübner, S.: Spatiotemporal reasoning for smart homes. In: Designing Smart Homes, pp. 16–34 (2006)
- Galton, A.: Causal reasoning for alert generation in smart homes. In: Designing Smart Homes, pp. 57–70 (2006)
- Cohn, A., Hazarika, S.: Qualitative spatial representation and reasoning: An overview. Fundam. Inf. 46(1-2), 1–29 (2001)
- Randell, D.A., Cui, Z., Cohn, A.: A spatial logic based on regions and connection. In: KR 1992. Principles of Knowledge Representation and Reasoning: Proceedings of the Third International Conference, pp. 165–176. Morgan Kaufmann, San Mateo (1992)
- Moratz, R.: Representing relative direction as a binary relation of oriented points. In: Brewka, G., Coradeschi, S., Perini, A., Traverso, P. (eds.) ECAI, Riva del Garda, Italy, pp. 407–411. IOS Press, Amsterdam (2006)
- Freksa, C.: Qualitative spatial reasoning. In: Mark, D., Frank, A. (eds.) Cognitive and linguistic aspects of geographic space, pp. 361–372. Kluwer, Dordrecht (1991)
- Freksa, C., Röhrig, R.: Dimensions of qualitative spatial reasoning. In: Carreté, N.P., Singh, M.G. (eds.) Proceedings of the III IMACS International Workshop on Qualitative Reasoning and Decision Technologies – QUARDET 1993, CIMNE Barcelona, pp. 483–492 (1993)
- Balbiani, P., Osmani, A.: A model for reasoning about topologic relations between cyclic intervals. In: Cohn, A.G., Giunchiglia, F., Selman, B. (eds.) KR 2000: Principles of Knowledge Representation and Reasoning, pp. 378–385. Morgan Kaufmann, San Francisco (2000)
- Freksa, C.: Using orientation information for qualitative spatial reasoning. In: Frank, A.U., Campari, I., Formentini, U. (eds.) Theories and methods of spatiotemporal reasoning in geographic space, pp. 162–178. Springer, Berlin (1992)

- Schlieder, C.: Reasoning about ordering. In: Kuhn, W., Frank, A.U. (eds.) COSIT 1995. LNCS, vol. 988, pp. 341–349. Springer, Heidelberg (1995)
- Freksa, C.: Conceptual neighborhood and its role in temporal and spatial reasoning. In: Singh, M.G., Travé-Massuyès, L. (eds.) Proceedings of the IMACS Workshop on Decision Support Systems and Qualitative Reasoning, pp. 181–187. North-Holland, Amsterdam (1991)
- Renz, J., Rauh, R., Knauff, M.: Towards cognitive adequacy of topological spatial relations. In: Spatial Cognition II, Integrating Abstract Theories, Empirical Studies, Formal Methods, and Practical Applications, pp. 184–197. Springer, Heidelberg (2000)
- Knauff, M., Rauh, R., Renz, J.: A cognitive assessment of topological spatial relations: Results from an empirical investigation. In: Frank, A.U. (ed.) COSIT 1997. LNCS, vol. 1329, pp. 193–206. Springer, Heidelberg (1997)
- 14. Clarke, B.L.: A calculus of individuals based on 'connection'. Notre Dame Journal of Formal Logic 22(3), 204–218 (1991)
- Clarke, B.L.: Individuals and points. Notre Dame Journal of Formal Logic 26(1), 61–75 (1985)
- Egenhofer, M.J., Franzosa, R.D.: Point Set Topological Relations. International Journal of Geographical Information Systems 5(2), 161–174 (1991)
- Dylla, F., Wallgrün, J.O.: Qualitative spatial reasoning with conceptual neighborhoods for agent control. Journal of Intelligent and Robotic Systems 48(1), 55–78 (2007)
- Lin, F.: Embracing causality in specifying the indirect effects of actions. In: IJCAI, pp. 1985–1993 (1995)
- Bhatt, M., Loke, S.: Modelling dynamic spatial systems in the situation calculus. Journal of Spatial Cognition and Computation: Special Issue on Spatio-Temporal Reasoning (to appear, 2008)
- Lin, F., Reiter, R.: State constraints revisited. Journal of Logic and Computation 4, 655–678 (1994)