Stochastic programming as a tool for emergency logistics in natural floods

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Abstract

Catastrophic events pose hard logistics chalenges, because the transportation and/or communication networks are damaged and supply chain capacities are affected. Under these circumstances, the demand patterns change in shape and magnitude. Thus the standard array of consumption typically forecasted by producers and distributors is no longer valid and hence standard logistics practices are unable to deliver goods and services on time at the right place. This article presents a modeling framework to assist decision makers in the planning stage of immediate assistance of natural disasters victims. The modeling framework gives an optimal inventory policy for emergency supplies and optimal fleet distribution. In this article, floods are the only possible events in the foreseeable future. It is possible to establish a stochastic process to represent the probabilistic occurrence of floods in different zones throughout a year.

The mathematical model that optimizes the inventory levels flows and vehicles allocation is a large size stochastic integer programming model. The model is (approximately) solved through sample average approximation. An example is provided.

1 Introduction

Most of the logistics systems are designed to operate under standard conditions, i.e., when the transportation and communication networks are fully operative, the suppliers are able to deliver what they are asked and the demand patterns fluctuate within (somewhat) known bounds, as well as availability of human resources and vehicles to distribute products and services from production sites to consumption points. Even under this scenario, the logistics strategy and operation are rather complex tasks both for the size of the instances to be solved and for the type of models to be solved when trying to optimize sensible variables such as inventory levels or sequences of vehicles stops under time constraints. However, this already complex situation becomes much more cumbersome when there is uncertainty on some (or all of the) system´s components. That is the case in the logistics of emergencies right after a natural disaster has occurred. Typically, there will be victims that need prompt attention in several dimensions: health care, food, water, safety, and childcare among others. However, the means to deliver the assistance may have been severely damaged by the natural disaster and hence the standard supply plans of the various industries involved in the provision of these goods do not hold in the disastrous scenario. Emergency logistics plans consider pre-positioning supplies and vehicles to reduce the travel times as well as transportation capacity; in addition, an optimal assignment of supplies and vehicles to a given location assures the minimization of total systems cost while at the same time satisfying time constraints.

Natural disasters fall under the category of low-probability high-consequence events, both in terms of human, animal and economic losses. In this article we focus only on disasters that exhibit a seasonal and/or spatial recognizable pattern that can be adequately represented by a (computationally) tractable stochastic process. That is the case of natural floods, which occur mostly during the rain seasons and affect mostly low lands or plain enclosed areas. Thus, their occurrence pattern can be represented by a certain space-time probability distribution model. Floods can be generated by a number of factors, where excessive rain ranks among the most common natural causes of floods in Chile, affecting mostly cattle and rural towns.

1.1 Objective

The aim of this article is to develop a modeling structure to assist the decision makers in tactical aspects of the emergency logistics after a flood occurrence, answering the following questions: What type and quantity of products should be kept in stock at each location and period within the area of interest and time horizon respectively? and, How to transport the supplies from the stocking facilities to the demand points at minimum cost and time?

1.2 Literature Review

In spite of the enormous relevance that emergency logistics has for the current society, the number of publications in this topic is considerably lower than that of commercial logistics. The most cited publication on emergency logistics, related to natural disasters, is [Ozdamar *et al.*, 2004], which focuses on operational decisions in natural disasters in general. [Fiedrich *et al.*, 2000], presents a study for strategic decisions in the case of earthquakes. [Chang *et al.*, 2007], develops a strategic model for the case of floods, considering different scenarios. The type of model developed in this article falls into the category of stochastic programming. In particular, we developed a probabilistic programming model. Recently, [Pagnoncelli *et al.*, 2009; Luedtke and Ahmed, 2008], present methods to solve real-case problems involving probabilistic programming modeling.

To the knowledge of the authors, at the time this article was written, no articles have been published with developments in probabilistic programming models applied to tactical decisions in emergency logistics.

2 Problem Description

2.1 Some Characteristics of a Natural Disaster

After the occurrence of a natural disaster, a number of negative impacts reach the population in the affected area. In the case of rural areas, where cattle raising is the main economic activity, the floods endangers the animal production as well as the crops used to feed the cattle, generating a deficit in supplies. Therefore, after a flood the cattle producers will demand forage, fresh water, nutritional supplements and medicines in larger amounts than they would normally do. The latter is an emergency demand, which must be satisfied as quickly as the transportation and communication networks allow. In this case the local authority is the the responsible for delivering the emergency supplies accurately and on time. These situations typically involve a minimum scale time of 4 seasons, a spatial span between 5 and 15 zones, at least 2 types of vehicle (e.g. dry and reefer) and at least 2 types of supplies (e.g. water and solid foods). From the tactical perspective, the emergency logistics should aim to maintain levels of inventory of (a few) urgently needed goods, sufficiently high to satisfy the expected demand for them after an event, but at the same time not too high to avoid costly excessive stock. That means that the inventory levels should guarantee the satisfaction of demand to a certain confidence level, i.e., to an acceptable degree of failure. For instance, a desired objective could be to find the levels of inventory that assures that demand will be satisfied at least in the 95% of the cases. Additionally, the demand satisfaction must not only be within the desired confidence level but also must be done at minimum cost, where the cost is any generalized distance that may include response time and resource consumption, among others.

The stochasticity inherent to the occurrence of floods makes the demand function to change abruptly from zero to a high level of consumption. This is a characteristic rarely seen in the consumption of standard goods (or goods under standard conditions) and hence the standard levels and location of stocks may not be able to fulfill the demand within acceptable time limits after a flood has stroke an area.

2.2 System Agents

Products Suppliers

These agents are willing to offer a set of products of potential demand. Finished products are stocked in different locations spread out within an area significantly larger than any affected zone. If an emergency occurs, some of the products would be retrieved from one or more depots in the amounts that a minimum cost flow program recommends. Note that prices may vary between producers as well as within the same producer across time periods.

Transportation Providers

Carriers offer the service of moving goods from the suppliers depots to the demand points. They operate with a fleet of vehicles that are either parked in different locations or moving between a large number of origins and destinations. Thus, within their whole fleet, it is possible to ascribe an available fleet per location during any given period. The transportation tariffs are charged per unit of freight per travelled distance. The tariffs may vary among different carriers as well as within the same carrier across time periods.

Vehicles within a fleet are not necessarily homogeneous. Certain products need a special type of vehicle (e.g. vaccines) and hence there is a compatibility matrix which shows acceptable vehicle-product pairs.

Demand Points

They correspond to geographic zones affected by the flood or specially designated loading/unloading facilities. These zones represent the demand for products (consumption) even though the actual consumption might take place in a different location. The magnitude of the demand at these zones is formed by aggregation of individual demands for all the affected cattle producers assigned to that particular zone.

3 Modeling the Problem

Both the time horizon and the geographic scope are defined a priori by the decision maker. The geographic area is subdivided into regions and the time horizon is divided into discrete periods. The demand for products that would be generated by a flood must be forecasted on this spatio-temporal grid. Those demand forecasts, along with some initial conditions, define an instance of the problem to be solved.

At any of the regions of the specified geographic partition, at a certain time period a flood may occur with a given probability. This probability is drawn from the history of flood records for similar seasons in the past.

For any given region and time period, the probability of flood occurrence can be estimated via a Bayesian Processor of Forecasts [Kelly and Krzysztofowicz, 1994] which maps both an a priori description of the uncertainty about flood occurrence and its magnitude, and a posterior description of uncertainty about flood occurrence and its magnitude, conditioned on a flood magnitude forecast. Note however, that the accuracy of these forecasts will largely depend on the type of phenomenon producing the flood. For instance, in forecasting hurricane-induced floods or when a severe thunderstorm is developing, there may be little uncertainty about the large amount of rainfall to be produceed, but there may be large uncertainty about its spatio-temporal location. This situation would imply significant uncertainty regarding the occurrence of rainfall over critical points in the study area within the reach of the storm.

Once the event strikes a populated area, there is an immediate generation of demand for certain basic products (e.g. fresh water and food for cattle). Then, the magnitude and composition of the demand will change probabilistically according to the initial conditions and the severity and scope of the emergency. While some of the initial conditions can be accurately known (such as number of cattle producers, inventory at hand, and operational characteristics of the potentially affected area), other conditions (such as the severity and duration of the event) cannot be predicted with acceptable degrees of accuracy. Therefore, it is necessary to fit a set of probability distribution functions to represent the magnitude of the demand and the possible scenarios that a particular realization of the event would generate.

On top of the uncertainty posed by the demand function, there is another source of uncertainty that must be taken into consideration: the capacity loss on the transportation network. Indeed, the networks normal conditions are affected not only by direct action of the catastrophic event but also for the impulsive behavior of users who may overcharge some of the arcs creating extra congestion with unnecessary trips that preclude the timely access of emergency teams. Additionally, the media exposure generates compulsive donations of unwanted items that may saturate the logistic system with the convergence of *materiel* that is both unnecessary and hard to handle.

3.1 Mathematical Formulation of the Main Problem

This section presents the formal modeling of the problem at hand, based on the conceptual description provided in the previous section.

General Definitions

The stochasticity of the demand vector imposes the definition of various probabilistic parameters:
p_i^t: Probability that a flood occurs at location i during period t.
P_i^t: Bernoulli random variable which takes value 1 with probability p_i^t.
$\rho p_{i,j}^{t_a,t_b}$: Correlation between $p_i^{t_a}$ and $p_j^{t_b}$, for locations i and j, and periods t_a and t_b.

The demand function for product p, at location i, during period t is defined as follows:

$$D_{ip}^t = \begin{cases} d_{ip}^t & \text{with probability } p_i^t \\ 0 & \text{with probbility } 1 - p_i^t \end{cases} \tag{1}$$

$$d_{ip}^t \sim F_{ip}^t(x) \tag{2}$$

$F_{ip}^t(x)$ is some probability distribution function over a real non-negative domain.

Analogous to the case of a flood, the demand for products may also exhibit significant correlation both in time and space. Accordingly, we define:
$\rho d_{i,j}^{t_a,t_b}$: Correlation between $d_{ip}^{t_a}$ and $d_{jq}^{t_b}$, for locations i and j, and periods t_a and t_b.

The Optimization Problem

The solution to this problem is aimed to assist the decision maker in tactical aspects of the emergency logistics after the flood occurrence. Thus, the model will answer the following questions:

1. What type and quantity of products (potential demand) should be kept in stock at each location and period within the area of interest and time horizon respectively?
2. How to transport the demanded products from the stocking facilities to the demand points at minimum (generalized) cost?

To answer these questions we need to establish the models parameters and variables:
I: Number of locations within the area of interest.
Φ: Set of all the potentially affected locations $i = \{1 \ldots I\}$.
P: Number of products.
Π: Set of all the potentially demanded products $p = \{1 \ldots P\}$.
T: Number of periods within the planning horizon.
Ψ: Set of all the periods $t = \{1 \ldots T\}$.
C: Number of vehicle classes.
Ω: Set of vehicle classes $c = \{1 \ldots C\}$.
D_{ip}^t: Demand for product (random variable) p, in location i, during period t.
d_{ip}^t: A realization of D_{ip}^t p, in location i, during period t.

u_c: Capacity of vehicle class c.
w_{pc}: Compatibility matrix product-class. Typical element (p, c) takes value 1 if product p, can be transported on the vehicle class c and 0 otherwise.
V_{ic}^t: Number of vehicle classes c, available in location i, at the beginning of period t.
L_{jp}: Inventory capacity at location j, for product p.
cv_{ijkp}^t: Cost of transporting one unit of product p, to the location k, originated from a supplier in location j, transported

on a vehicle available in location i, during period t.
cl^t_{ijc}: Cost of relocating a vehicle class c, sent from location i to location j, during period t.
ci^t_{jp}: Unitary inventory cost for product p, stocked in a depot located in j, during perod t.
α: Level of confidence; at least $100(1-\alpha)\%$ of the demand must be satisfied.

3.2 Variables

The following are the modeling variables:
x^t_{ijkp}: Flow of product p, sent to location k, originated from a supplier in location j, transported by a vehicle available in location i, during period t.
y^t_{ijc}: Flow of vehicle class$c$, relocated from location $i$to location j, during period t.
I^t_{jp}: Inventory of product p, kept in depot located in j, at the beginning of period t.

With the parameters and variables previously defined, the following mixed integer programming model is presented:

$$\min \sum_{t\in\Psi}\sum_{p\in\Pi}\sum_{k\in\Phi}\sum_{j\in\Phi}\sum_{i\in\Phi} cv^t_{ijkp}x^t_{ijkp} + \sum_{c\in\Omega}\sum_{j\in\Phi}\sum_{i\in\Phi} cl^t_{ijc}y^t_{ijc} + \sum_{t\in\Psi}\sum_{p\in\Pi}\sum_{j\in\Phi} ci^t_{jp}I^t_{jp} \quad (3)$$

$$P\left\{\sum_{j\in\Phi}\sum_{i\in\Phi} x^t_{ijkp} \geq D^t_{kp} \quad \forall k\in\Phi,\, p\in\Pi,\, t\in\Psi\right\} \geq \alpha \quad (4)$$

$$\sum_{p\in\Pi}\sum_{k\in\Phi}\sum_{j\in\Phi} w_{pc}x^t_{ijkp} \leq u_c \sum_{j\in\Phi} y^t_{ijc} \quad \forall i\in\Phi,\, t\in\Psi,\, c\in\Omega \quad (5)$$

$$\sum_{j\in\Phi} y^t_{ijc} \leq V^t_{ic} \quad \forall i\in\Phi,\, t\in\Psi,\, c\in\Omega \quad (6)$$

$$\sum_{k\in\Phi}\sum_{i\in\Phi} x^t_{ijkp} \leq I^t_{jp} \quad \forall j\in\Phi,\, p\in\Pi,\, t\in\Psi \quad (7)$$

$$I^t_{jp} \leq L_{jp} \quad \forall j\in\Phi,\, p\in\Pi,\, t\in\Psi \quad (8)$$

$$x^t_{ijkp}, I^t_{jp} \in \mathbb{R}^+ \quad \forall i\in\Phi,\, j\in\Phi,\, k\in\Phi,\, p\in\Pi,\, t\in\Psi \quad (9)$$

$$y^t_{ijc} \in \mathbb{Z}^+ \quad \forall i\in\Phi,\, j\in\Phi, c\in\Omega,\, t\in\Psi \quad (10)$$

The objective function (3) has three terms: the cost of transporting products to the disaster areas; the cost of moving vehicles between depots, shipping origins and shipping destinations; the cost of carrying prepositioned inventory. The set of constraints (4) ensures the demand satisfaction. The constraint set (5) restricts the available transportation capacity for each period and site. The constraint set (6) controls that the number of transferred vehicles does not exceed its availability for each period. The constraint set (7) ensures that the flows of products do not exceed the initial inventory available to the suppliers. Constraint set (8) controls that the initial stock level does not exceed the inventory capacity. Finally, the constraint sets (9) and (10) impose integrality and non negativity. If it was not for the cumbersome constraint (4), this model could be solved through standard mixed integer programming methods. Unfortunately, the shape of (4) precludes that approach. Consequently, another technique must be used to incorporate the inherent stochasticity. One such approach is the Sample Average Approximation method described in the following section.

4 Sample Average Approximation

In this section, a Sample Average Approximation (SAA) scheme is implemented (as presented in [Pagnoncelli *et al.*, 2009]). The general idea is to replace the constraint (4) by a (larger) set of new deterministic constraints that approximate the stochastic model; it will be a larger model but simpler, because it does not consider stochasticity at all, nevertheless it must be solved repeatedly to simulate the probability of event occurrence considered in the original model, which increases the complexity of the problem. The SAA scheme implemented in this research solves a series of scenarios consisting in different instances of the modified optimization model (expressions 11 to 20), each instance corresponding to a realization of a Monte Carlo simulation of demands (expression 1). The solution of this series of instances of the modified optimization model provides a lower and upper bound to the solution of the original problem.

Let z^{tn}_{kp} be binary variables that measure the number of times that a demand constraint is not satisfied. Thus, the following modified optimization model is defined for the generated samples.

$$\min \sum_{n=1}^{N}\sum_{t\in\Psi}\sum_{p\in\Pi}\sum_{k\in\Phi}\sum_{j\in\Phi}\sum_{i\in\Phi} cv^t_{ijkp}x^{tn}_{ijkp} + \sum_{n=1}^{N}\sum_{c\in\Omega}\sum_{j\in\Phi}\sum_{i\in\Phi} cl^t_{ijc}y^{tn}_{ijc} + \sum_{t\in\Psi}\sum_{p\in\Pi}\sum_{j\in\Phi} ci^t_{jp}I^t_{jp} \quad (11)$$

s.a.

$$\sum_{j\in\Phi}\sum_{i\in\Phi} x^{tn}_{ijkp} + z^{tn}_{kp}D^{tn}_{kp} \geq D^{tn}_{kp} \quad \forall k\in\Phi,\, p\in\Pi,\, t\in\Psi,\, n=1,\ldots,N \quad (12)$$

$$\sum_{p\in\Pi}\sum_{k\in\Phi}\sum_{j\in\Phi} w_{pc}x^{tn}_{ijkp} \leq u_c \sum_{j\in\Phi} y^{tn}_{ijc} \quad \forall i\in\Phi,\, t\in\Psi,\, c\in\Omega,\, n=1,\ldots,N \quad (13)$$

$$\sum_{j\in\Phi} y^{tn}_{ijc} \leq V^t_{ic} \quad \forall i\in\Phi,\, t\in\Psi,\, c\in\Omega,\, n=1,\ldots,N \quad (14)$$

$$\sum_{k\in\Phi}\sum_{i\in\Phi} x^{tn}_{ijkp} \leq I^t_{jp} \quad \forall j\in\Phi,\, p\in\Pi,\, t\in\Psi,\, n=1,\ldots,N \quad (15)$$

$$I_{jp}^{t} \leq L_{jp} \quad \forall j \in \Phi,\, p \in \Pi,\, t \in \Psi \tag{16}$$

$$\sum_{n=1}^{N} \sum_{t\in\Psi} \sum_{p\in\Pi} \sum_{k\in\Phi} z_{kp}^{tn} \leq N(1-\gamma) \tag{17}$$

$$x_{ijkp}^{tn}, I_{jp}^{t} \in \mathbb{R}^{+} \quad \forall i \in \Phi,\, j \in \Phi,\, k \in \Phi,\, p \in \Pi,\, t \in \Psi,\, n = 1, \ldots, N \tag{18}$$

$$y_{ijc}^{tn} \in \mathbb{Z}^{+} \quad \forall i \in \Phi,\, j \in \Phi, c \in \Omega,\, t \in \Psi,\, n = 1, \ldots, N \tag{19}$$

$$z_{kp}^{tn} \in \{0, 1\} \quad \forall k \in \Phi,\, p \in \Pi,\, t \in \Psi,\, n = 1, \ldots, N \tag{20}$$

where, the upper index índice n is the sample number, N is the sample size and γ is the desired level of accuracy to solve the approximated problem. this level of service is not necessarily identical to the original α level originally defined. Constraint (17), allows that the number of times that the demand satisfaction constraint (12) is violated , does not exceeds $1-\gamma$.

4.1 Lower Bound

To obtain a lower bound a sample average approximation scheme is applied (see [Pagnoncelli *et al.*, 2009]). The first step is to find two integer numbers M and N such that:

$$\theta_N := \sum_{i=0}^{\lfloor (1-\gamma)N \rfloor} \binom{N}{i} \alpha^{i} \left(1-\alpha\right)^{N-i} \tag{21}$$

and L being an integer number such that:

$$\sum_{i=0}^{L-1} \binom{M}{i} \theta_N^{i} \left(1-\theta_N\right)^{M-i} \leq 1-\beta \tag{22}$$

Then, a set of M independent samples must be generated: $D_{kp}^{t1m}, \ldots, D_{kp}^{tNm}, m = 1, \ldots, M$ each one of size N.

For each generated sample, the above modified optimization problem must be solved.

The optimal solution for each sample, called $\hat{\theta}_N^{m}$, $m = 1, \ldots M$, must be arranged in non decreasing order, $\hat{\theta}_N^{(1)}, \ldots, \hat{\theta}_N^{(M)}$, where $\hat{\theta}_N^{(i)}$ is the i-th smallest value.

Finally, the value $\hat{\theta}_N^{(L)}$ will be a lower bound for the optimal solutyion of the original problem, with a significance level of at least β.

4.2 Upper Bound

To obtain an upper bound, the method put forward by [Luedtke and Ahmed, 2008] will be applied. One of the findings of that article is the size N of a sample to guarantee that the solution of the modified optimization problem be in fact a feasible solution for the original problem, with a significance level of β. The latter is obtained as follows:

$$N \geq \frac{2}{\left(\alpha-\gamma\right)^{2}} \log\left(\frac{1}{1-\beta}\right) + \frac{2m}{\left(\alpha-\gamma\right)^{2}} \log\left\lceil \frac{2DL}{\alpha-\gamma} \right\rceil \tag{23}$$

This result gives a theoretical guide for the search of the sample size N. However, the problem size (given the obtained value of N) could be prohibitively large.

An alternative to this method is to solve the modified optimization problem with a smaller value of N and then checking (a porteriori) the fulfilment of the stochastic constraint.

This a porteriori checking can be done by using a sample of size N', and then for the samples $D_{kp}^{t1}, \ldots, D_{kp}^{tN'}$ counting the number of times that this expression holds: $\sum_{j\in\Phi} \sum_{i\in\Phi} x_{ijkp}^{tn} \geq D_{kp}^{tn}$.

The upper bound will be the objective's function value, corresponding to the solution with the highest value within all the feasibel solutions, once the a posteriori checking was performed (see [Luedtke and Ahmed, 2008]).

5 Numerical Example

In this section a numerical instance is presented, applying the above described methods to obtain both the lower and upper bounds.

The example consists of an instance with six locations, four periods, two products and two types of vehicle. Each location faces an iid lognormally distributed demand with mean 100 and a standard deviation of 10. For each period, the probability of a natural flood is assumed to be 0.2, with spatial and temporal correlations generated randomly.

5.1 Lower Bound

Using the methodology presented in section (4.1), the relevant parameters were found: $M = 172$, $N = 20$, $L = 12$, considering the following values: $\alpha = 0.9$, $\beta = 0.99$, $\gamma = 1$.

Using CPLEX 12, the modified optimization problem (presented in section 4) was solved $M = 172$ times. Each instance consisted of 960 binary variables, 5,760 integer variables and 34,608 continuous variables. Demands were generated with a Monte Carlo simulation routine using the parameters given above.
The 172 obtained solutions were then sorted in a non-decreasing order, obtaining a lower bound in $L = 12$, which value corresponds to $47,392$.

5.2 Upper Bound

To obtain the upper bound, we considered the values $N = 30$ and $\gamma = 0.95$, obtaining a feasible solution, which was tested a posteriori, considering 100 randomly generated demands. The instance consisted in 1,440 binary variables, 8,640 integer variables and 51,888 continuous variables.
The solution found was feasible in all of the 100 considered demands, which is a successful result given the high level of confidence set initially: $\alpha = 0.9$. The optimal value for each one of the 100 instances is the following. Minimum value = $52,989$; Maximum Value = $67,124$; Average = $55,991.6$; Standard deviation = $4,601.4$.

Thus, the upper bound to the original problem corresponds to $67,124$.

5.3 Results Analysis

The difference between both bounds is 41.6%. This gap is larger than that of other published results [Luedtke and Ahmed, 2008]. However, the type of problem addressed in this case is rather different from those previously tackled in the published literature. Indeed, natural floods are low probability-high consequence events, which means that the stochastic parameters exhibit a high variability when compared to other problems; in this case the coefficient of variation is 2, which is considerably higher than those in the reviewed literature (between 0.1 and 0.5 depending on the case).

Even though there is no formal indication about how conservative both bounds are, we can conclude, with high level of confidence, that the solution found for the upper bound will satisfy the emergency demands with the desired significance level. This is, of course, a highly valuable input to the decision makers, in spite of not knowing the exact value of the optimal objective function.

6 Conclusions

The occurrence of a flood is an event of low probability but often of high consequences in terms of material costs and sometimes human or animal losses. These characteristics (along with the size of realistic instances) make that phenomenon unsuitable to be modeled with standard mathematical programming techniques, which assume deterministic parameters.

In this article, the authors presented a stochastic programming model to represent tactical decisions in the logistics of emergencies after the occurrence of a natural flood. The model attempts to find the optimal levels of inventory in different locations as well as the flows of materiel from these locations to the affected areas. The model also gives the fleet size that must be available at each location to distribute an array of different types of products (which may need different types of vehicle to be delivered). In addition, the authors proposed a methodology to find an approximation to the optimal solution of that model. The approximation is found through the application of a recently published scheme of sample average approximation.

The proposed model and solution approach are general in the sense that no specific probability distribution function is assumed (or necessary) and the pattern of spatial and/or temporal correlations can be totally general. The applicability of this modeling structure is conditioned on the capacity of the modeler to generate spatiotemporal patterns similar to those observed in the real scenario as well as the capacity to handle the large size of the resulting instances to be solved.

The modeling structure was applied to a test instance with six locations, two prodcuts and four periods. The spatial and temporal correlations between flood-occurrence probabilities were generated randomly. Through the described methodologies upper and lower bounds were found for the original optimization problem. The gap between both bounds was relatively large (41.6%). However, the solution for the upper bound satisfies the emergency demands with the desired level of confidence, which is a very useful result for the decision makers under the preassure of delivering emergency aid in extreme events.

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