LINGUISTIC DESCRIPTION OF HUMAN JUDGMENTS IN EXPERT SYSTEMS AND IN THE 'SOFT' SCIENCES

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A fuzzy-linguistic tool for the representation and analysis of human judgments in expert systems and in the 'soft' sciences is presented. The approach is motivated by fundamental differences between measurements as they are performed in the 'hard' sciences and judgments which form the basis for decisions in the 'soft' sciences. These differences suggest a representation for human judgments which preserves their fuzziness instead of a representation in terms of the "measurement and error" paradigm used in the hard sciences. The paper explains why such a representation not only is more natural to use but also yields more reliable results. Finally, the interactive construction of semantic representations of linguistic descriptors from examples is discussed.

KEYWORDS: linguistic descriptors, soft data representation, possibility distributions, cognitive modelling, information granularity

1 INTRODUCTION

Research and expert areas like psychology, medicine, jurisdiction, etc. deal with 'soft' data [COLLINS et al., 1975; SKALA, 1978; ZADEH, 1979b]. These data result from a different type of interpretation process than 'hard' data in classical physics, chemistry, or engineering. Nevertheless, the same research methodology is being used to process data of both types. The reason for this must be an implicit assumption that both kinds of data are of the same nature, i.e., have the same conceptual framework, and therefore should be analyzed by the same methods.

This paper compares the concepts 'data-driven measurement' and 'goal-driven judgment' [NORMAN & RUMELHART, 1975; NILSSON, 1980] by investigating their underlying processes. Occasionally, these notions will be abbreviated in the following by 'measurement' and 'judgment', respectively. The paper argues that observation data may have fundamentally different semantics depending on the structure of their interpretation processes and should be represented and interpreted accordingly. While data-driven processes yield point data which are to be interpreted with certain error tolerances, goal-driven interpretation processes yield possibility distributions [ZADEH, 1978]

which can be constructed such that they are free of errors. It may not be meaningful to think of an error of a soft judgment the same way as of an error of a hard measurement.

By developing adequate representations of observations or judgments we may be able to take advantage of their inherent properties, specifically imprecision, variable fuzziness, and resolution [1]. These properties appear disadvantageous in the "measurement and error" paradigm which is conventionally used for the interpretation of observations [e.g. GULLIKSEN & MESSICK, 1960]. A fuzzy-linguistic representation terms of simplified possibility ributions [FREKSA, 1981] is i n distributions proposed. This representation is suited type of operations that for the observations are suited for in natural systems: communication, cognition comparison, and judgment.

We will point to empirical results which suggest that more natural (specifically: linguistic) representations of cognitive observations yield more informative and reliable interpretations than traditional arithmomorphic representations. We will demonstrate how semantic representations of linguistic descriptors [2] for cognitive observations can be constructed interactively from examples.

DATA-DRIVEN AND GOAL-DRIVEN OBSER-VATIONS

We will contrast two extreme examples of a purely data-driven and of a purely goal-driven observation process for our analysis. In these examples, we consider some phenomenon that is to be observed by some device. In the first case, this device is a measuring instrument, in the second case, it is a human observer.

2.1 Examples

EXAMPLE 1: Measurement process in the hard sciences.

Consider the function of a voltmeter. The physical dimension "voltage" may be interpreted in terms of electrical current that flows in a given conductor. The current causes a magnetic field which results in a physical force which displaces the needle of a voltmeter.

The displacement of the needle is an indication of the voltage at the instrument. The instrument yields a point value, i.e., the needle points at one particular position for a given voltage. A physical model relates each needle position to an input voltage at the meter. The position value of the needle therefore can be interpreted as an indication of the voltage. It must be considered an approximation of the actual voltage since certain factors influencing the correspondence between voltage and needle displacement have been ignored in the model (e.g. mechanical friction in the instrument, influence of magnetic field of the earth, etc.). The effect of these factors, however, can be estimated and the error range of the approximation can be determined.

EXAMPLE 2: Judgment process in the soft sciences.

Now consider psychological rating of a continuous quantity, say the degree of well-being of a person. This quantity is not functionally determined in terms of well-defined parameters as in the case of the voltmeter. Nevertheless, no one would argue that we have a definite perception of degree of well-being both for ourselves and for other beings. In certain situations we may be able to distinguish minute differences in well-being reliably, in other situations we can make only coarse distinctions. The outcome of an observation may be a linguistic description like "today she feels better than yesterday", "this

feels good", "this is slightly worse", etc.

The description of the observation is an indication of the degree of well-being of the observed person. The resolution (or relative precision [1]) of the observation depends not only on the observer (the instrument) but also on the information on which his judgment is based. The observation process yields a restricted range of possible states of well-being in form of a linguistic description. A heuristic model relates each description to a range of possible states. Although certain indicators for the state of well-being could not be taken into account the description may be a correct indication of the person's well-being; additional could have refined the state of information answer and thus could have further restricted the range of possible states of well-being.

We may view human judgments as cognitive processes which depend on limited data and resources — input information, processing power, memory — and thus may operate with fluctuating resolution [NORMAN & BOBROW, 1975].

2.2 Analysis

The two examples exhibit two different processes for feature evaluation. Physical measuring devices like voltmeters generate measurement values by physical analogy or simulation and yield point values corresponding to the single values they simulate. The measurement is a purely data-driven process [LINDSAY & NORMAN, 1977].

Resource-limited cognitive observation processes are goal-driven interpretation processes and do not generate values by functional simulation; they interpret sensory data by collecting evidence for a judgment. The more evidence they collect the finer (preciser) their judgment. In case of determining the state of well-being, a cognitive observation process initially (without any evidence) would judge "I don't know" which means all states of well-being are considered possible. The more information the observer gathers during the observation process, the more he may be able to refine his answer by restricting the range of possibilities down to a level at which no helpful information can be gathered which would modify or refine the judgment [3].

The difference in the type of outcome is a result of the different direction of

information processing: the data-driven process synthesizes a result from basic entities; the goal-driven process decomposes an undifferentiated whole into more specific, but still coarse components. In a sense, the final outcome of the observation process is atomic, i.e., it cannot be refined — at least not by this particular observer in the particular situation. The precision of this observation should not determine the resolution of the representing scale, however, since other observers and/or other situations may allow for a finer judgment. The observation may be said to correspond to a 'granule' rather than to a point on a fine scale [ZADEH, 1979a].

Since the interpretation process does not yield a single point value but a range of possible values, it is not meaningful to speak of a judgment error in the sense of a measurement error because of the incompleteness of the utilized information. Data-driven measurements per se may be precise but erroneous while goal-driven judgments may be correct despite (or: due to) their inherent imprecision. They only become incorrect if the observed feature value is outside the scope of values represented by the descriptor.

To round off the comparison with data-driven measuring process, cognitive judgment process outlined above would correspond to a change of shape of the needle of the measuring device: initially the needle would cover the entire range of the scale of the meter ("don't know the voltage"). During observation process, the needle would become narrower, restricting the possibilities of the actual voltage at the meter. This cannot happen in analogue simulation processes since certain information is required to give a meaningful reading and additional information is not helpful for refining the measurement. The effect achieved by narrowing the needle can be achieved by widening the scale, of course. In practice, voltage measuring processes are frequently supervised by a cognitive process which selects the scale: if we do not know the order of magnitude of the voltage to be measured, we will first select the largest meter range to obtain an approximate reading. Then we will restrict the range and expand the scale to obtain a more precise reading.

2.3 Summary

In summary, a physical measurement obtained by an analogue simulation

process yields a point value which may be interpreted as an approximation to some actual value, while a goal-driven cognitive interpretation process yields a range of possible point values. The precision of this interpretation is limited by the resources available to the judgment process. In view of this fact it seems absurd to force imprecise observations into precise representations and in doing so, adding errors to the results. It appears desirable to represent observations at the level of precision that has been obtained by the underlying observation process and to represent them error-free.

3. COGNITIVE ASPECTS OF OBSERVATIONS

If we want to represent observations of human observers, we should consider the cognitive process which leads from the perception to the description by which the observation is communicated. We can view the process of observing, describing, and interpreting a phenomenon as a chain of transformations on cognitive representations of the observed phenomenon.

3.1 Chain of cognitive transformations

object

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mental representation
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Verbal description
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formal representation
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interpretation

Fig. 1

We call the observable phenomenon the "object". The object is an entity whose feature values (e.g. color, size, shape) can be viewed as being precise. In other words, there is no imprecision in the objects themselves [RUSSELL, 1923]. Imprecision and fuzziness enters when the object is perceived or represented.

If we assume a goal-driven (interpretative) rather than a data-driven (generative) feature transformation process, computational effort grows with the number of possible outcomes. If the possible outcomes were densly distribu-

ted point values, the effort would be To be able to represent an enormous. object with reasonable effort using limited resources, the information must be reduced. It will not be possible to recover the features completely from their description alone. However, if the description serves to select an object from a given context (i.e., the possible outcomes are not densly distributed), 'good' feature descriptors need not represent the features object precisely as possible, they only need to be able to discriminate between feature values that exist in the given context. The context then provides the additional information for object identification.

3.2 Consequences for the representation

The first transformation occurs in the perception process: due to limited resolution of any perception system only a limited number of feature values can be distinguished. Thus, a perceived object feature cannot be adequately represented by a point on a high-resolution feature scale but rather by an interval which takes into account the granularity of the observation IZADEM, 1979al.

Limited resolution also implies that the "observation granules", i.e. the intervals which represent the observations, cannot have sharp boundaries. Therefore, the observation granules should be represented by fuzzy rather than by crisp intervals. To simplify the specification of such intervals, we may

define fuzzy categories for the classification of the observations.

As a consequence of the granularity -which is inherent in any observation of phenomena in continuous feature spaces - it is impossible to reliably classify arbitrary observations into single categories, whether the categories are crisp or fuzzy. Observation granules may overlap the boundary of a category. For this reason, their representation must allow for simultaneous assignment of several neighboring categories if all object feature values which consistent with the observation are to be accounted for.

If we use crisp categories, the feature resolution cannot exceed half the resolution of the category itself (compare sampling theorem). If we use fuzzy categories, however, we can regain precision by a more differentiated assignment of categories. For example, we may be able to indicate whether a given category is

- 1 definitely applicable,
- 2. definitely not applicable, or
- neither definitely applicable nor definitely inapplicable.

In this way, the position of the observation granule on the feature scale can be recovered more fully. Fig. 2 depicts the classification of an observation granule into crisp and into fuzzy categories.

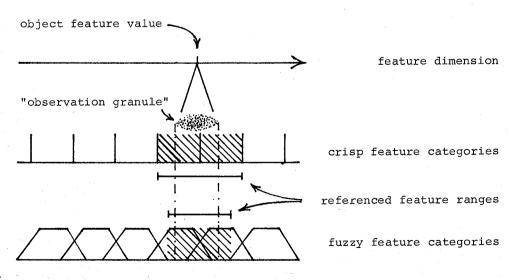


Fig. 2

3.3 <u>Transformations</u> in <u>natural</u> systems

We return to Fig. 1 which shows the description generation process as a chain of transformations on information about object features. The first transformation shown (from "object" to "percept") corresponds to the segmentation of input information for the perception system, e.g. into visual or auditory channels. This transformation has been studied anatomically and neurophysiologically for various senses in various animals and a great deal is known about its properties [e.g. LETTVIN et al., 1959].

The second transformation (from "percept" to "mental description") corresponds to the mental organization of this information in memory. Presently, this transformation is the subject of much speculation [e.g. MINSKY, 1980], but very little is actually known.

The third transformation (from "mental description" to "verbal description") is of particular interest in this paper. Since we do not know the structure of the mental description, we cannot evaluate this transformation directly. However, we can study some of its properties by analyzing the correspondence between different descriptions of features of the observed object. In particular, we can examine the consistency between various verbal descriptions of the same features. The degree of inconsistency between those descriptions gives us an indication of the distortion introduced into the information during the cognitive perception process. The verbalizations which introduce the least amount of distortion are suited best to recover the observed object feature.

4. COMPARISON OF THREE REPRESENTATION SCHEMES

4.1 Goal

To illustrate different degrees of distortions which we may get by different representation schemes, we describe a simple empirical experiment which will be reported in detail elsewhere. In this experiment we compared three cognitively processed description types with feature measurements, namely of the height of persons [4]. The experiment consisted of three independent parts. 23 medical students were asked to judge the height of their colleagues by various methods. The

"objects" of observation were asked in non-systematic order to present themselves in front of the observers (the remaining 22 students). The observers noted their judgment of the height on a prepared sheet containing the names of the "objects".

4.2 Experiment

PART 1: The observers were asked to estimate the height of the objects in centimeters.

PART 2: The observers were asked to indicate four values (in centimeters) to express their judgment:

- the minimum height value which they considered possible as an estimate of the actual value;
- a pair of two height values to specify a range in which they expected the actual height value to be;
- the maximum height value which they considered possible as an estimate of the actual value.

PART 3: The observers were asked to select from an ordered vocabulary of seven terms (very small, small, rather small, medium, rather tall, tall, very tall), all those terms which described appropriately the actual height of the "objects". In case they selected several terms they could indicate preferences of some terms over others.

After completion of these judgments, the height of the objects was measured (in centimeters) conventionally.

4.3 Results

In part 1 of the experiment, the maximum mis-judgments of the observers ranged between 8 and 13 centimeters. The mis-judgments were non-systematic, i.e., all observers sometimes overestimated and sometimes underestimated the actual height in a given vicinity of height values.

Part 2 of the experiment did not yield better results. Most observers apparently did not have a good feeling for the accuracy and precision of their own centimeter judgments. Frequently, the observers overestimated their ability by indicating too narrow possibility ranges for the height value. In many instances, the measured height value fell into the ranges which were considered entirely impossible by the

observers.

Part 3 of the experiment avoided the problem of erroneous label assignment by letting the labels denote just what the observer used it for. But surprisingly, the ranges of referenced height values rarely became larger than for the centimeter labels and frequently the specificity was even higher. In addition, by multiple assignment of labels with overlapping reference ranges the specificity of a description could be further increased.

The foregoing experiment shows that human judgment ability may be better than it appears from numerically expressed judgments. If we allow observations to be expressed in a more appropriate language, higher precision may be obtained. At first glance, this may appear contradictory, since a centimeter scale suggests higher resolution than a set of seven linguistic labels for a range of about 40 centimeters. However, by permitting simultaneous assignment of several labels we increase rather than decrease the potential of resolution. In addition, if these labels are applied in a more consistent manner than the centimeter values, they may resolve specific height values particularly well.

For interpretation of these results it may be helpful to consider again the information transformation model depicted in Fig. 1. The distortion of information introduced in the third transformation (mental description to verbal description) appears to depend on the type of verbalization that is used. Numerical verbalization seems to let rather precise observations appear imprecise, in many observers, while linguistic verbalizations seem to preserve more information from these observers.

We can explain this phenomenon by 'cognitive distance'. A linguistic representation of an observation may require a less complicated transformation than a numerical representation, and therefore less distortion may be introduced in the former than in the latter [PALMER, 1981]. We could say that the linguistic representation is "cognitively closer" to the mental description than the numerical representation. This theory is supported by the impression that the observers can make linguistic Judgments more spontaneously than numerical Judgments.

5. REPRESENTATION OF SOFT OBSERVATIONS

5.1 Desiderata

On the basis of the arguments brought forward in the foregoing sections we develop a representation system for "soft observations" which should have the following properties:

- the resolution of the representation should be flexible to account for varying precision of individual observations;
- the boundaries of the representing objects should not necessarily be sharp and should be allowed to overlap with other representing objects;
- comparison between different levels of resolution of representation should be possible;
- comparison between subjective observations of different observers should be possible;
- the representation should have a small 'cognitive distance' to the observation;
- it should be possible to construct representing objects empirically rather than from theoretical considerations.

5.2 Simplified possibility distributions

Feature descriptors contain, for the most part, possibilistic information [GAINES & KOHOUT, 1975; ZADEH, 1978]. Thus, a low-resolution descriptor allows for many possible interpretations while a high-resolution descriptor restricts the range of possible interpretations. Imprecise observations correspond to a wide range of possibilities wide range of possibilities while precise observations correspond to a comparatively narrow range of possibilities. To allow for unsharp boundaries, we express the observations hп quadruples representing simplified fuzzy possibility distributions [FREKSA, 1981]. These possibility distributions semantic representations of are linguistic descriptors and can be graphically depicted as shown in Fig. 3:

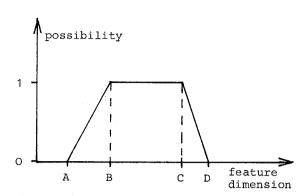


Fig. 3

The interpretation of the quadruple (A,B,C,D) is: it is entirely possible that the actual feature value observed is in the range [B,C]; it may be possible that the actual value is in the ranges [A,B] or [C,D], but more easily closer to [B,C] than further away; an actual value outside of [A,D] is incompatible with the observation. [B,C] is called "core", [A,B] and [C,D] are called "penumbra" of the possibility distribution.

5.3 <u>Construction of semantic</u> representations

Our demand for a 'cognitively close' representation of descriptors requires that the human observer determines the applicability of a descriptor according to his intuition-guided judgment. This is in contrast to the possibility of defining the meaning of descriptors by external criteria as is usually done in applications of fuzzy sets [ZADEH, 1970]

A consequence of this demand is that semantic representation of descriptors will be subjective; each observer may choose his own ranges of applicability of a descriptor. At first glance, this may appear disadvantageous. However, as the experiment above suggests, an external definition would only serve to introduce errors into the observation. Therefore the apparent simplifications for comparing observations would be paid for by loss in informativity. For this reason we prefer subjective semantic representations which necessitate semantic translations for inter-observer comparisons. Such translations are not very difficult to perform with the representation scheme we have chosen.

The construction of a repertoire of semantic representations for linguistic descriptors is done in the following way:

- the observer selects a set of linguistic labels which allows for referencing all possible values of the feature dimension to be described;
- the repertoire of linguistic labels is arranged linearily or hierarchically in accordance with their relative meaning in the given feature dimensions;
- 3. a set of examples containing a representative variety of feature values in the given feature dimension is presented to the observer. The observer marks all linguistic labels which definitely apply to the example feature value with "yes" and the labels which definitely do not apply with "no". The labels which have not been marked may be applicable, but to a lesser extent than the ones marked "yes";
- 4. from the data thus obtained we construct simplified possibility distributions by arranging the example objects according to their feature values (using the same criterion to which the linguistic labels had been arranged). These values form the domain for the assignment of possibility values;
- 5. finally, we assign to a given label the possibility value "yes" to the range of examples in which the given label was marked "yes" for all examples and the possibility value "yes" to the ranges in which the given label was marked "no" for all examples. The break-off points between the regions with possibility value "no" and "yes" are connected by some continuous, strongly monotonic function to indicate that the possibility of label assignment increases the closer one gets to the region with possibility assignment "yes".

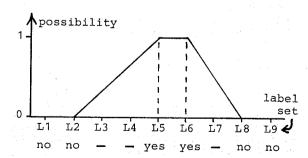


Fig. 4

CONCLUSIONS

We have described some characteristics of human judgments, investigated their causes, and proposed a method for representing judgments such that we can take advantage of these characteristics. In this last section we will discuss applications of the system and extensions being planned. Finally we will make a short comparison with related approaches.

6.1 Applications

The meaning representation system described appears useful for a variety of applications. First, it is a method for representing qualitative information and may aid in developing a semantic theory of descriptors as well as a theory of acquisition of imprecise concepts. Second, it is a tool for representing complex data which can be characterized best by humans.

At present, we use the approach for representing psychological judgments in clinical diagnosis. The method is suitable for semantic interpretation of subjective judgments both by graphical methods and by an interactive computer program. The semantic interpretation leads to an objectivization of descriptions and may be useful for person — machine communication as well as for person — person communication.

6.2 Interactive meaning adaptation

In an associated project [AGUILAR-MARTIN & LOPEZ DE MANTARAS, 1982; FREKSA & LOPEZ DE MANTARAS, 1982] learning algorithms for interactive meaning adaptation of linguistic descriptors by means of examples are being developed. In conjunction with a classification program, these algorithms will be helpful for custom tailoring individual descriptor languages for specific context situations. With the aid of these algorithms it will be no longer necessary that different observers use the same vocabulary for their descriptions. The denotation of the different descriptor labels is determined by the algorithms during a learning phase.

6.3 Comparison with other approaches

The representation scheme presented combines advantageous features of a variety of scaling and representation methods [LINDSAY & NORMAN, 1977]:

- ease of label association of a nominal scale;
- ease of comparison of an ordinal scale;
- meaning consistency of absolute scale;
- non-numerical scaling of direct scale;
- intuitiveness of cross-modality matching;
- naturalness of natural language approach.

From a methodological point of view, fuzzy classification of perceptually processed information is preferable over crisp classification. Conventional fuzzy set approaches use a priori definitions of fuzzy sets which apply universally for all observers. The advantage of our approach is increased consistency due to subjectiveness and small cognitive distance of the representation.

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NOTES

[1] We use the term 'precision' as a measure of relative specificity: a descriptor is called 'precise' if it has only one denotation in the given context, and 'imprecise' if it allows for a variety of interpretations.

A descriptor is called 'fuzzy' if its applicability varies gradually with respect to a variation of the reference feature and 'crisp' if it is either fully applicable or fully inapplicable.

'<u>Variable feature resolution</u>' refers to flexibility in the reference range of a feature descriptor: depending on the particular situation context, the same linguistic descriptor may be taken to refer to a more specific range of feature values.

- For more details, see FREKSA (1981).
- [2] We use the term 'descriptor' if we refer to a single feature dimension and the term 'description' if we refer to a general feature space.
- [3] This is an idealized judgment process in which no premature conclusion is drawn.
- [4] Height was chosen 1) since it can easily be measured, 2) since most people have some experience in judging height, and 3) since height has served as standard example in fuzzy set theory [e.g. ZADEH, 1978; BALDWIN, 1979] and has been used in other psychological studies [e.g. HERSH & CARAMAZZA, 1976].

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