A Relative Orientation Algebra with Adjustable Granularity

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Abstract

The granularity of spatial calculi and the resulting mathematical properties have always been a major question in solving spatial tasks qualitatively. In this paper we present the Oriented Point Relation Algebra ($\mathcal{OPRA}_m$), a new orientation calculus with adjustable granularity. Since our calculus is a relation algebra in the sense of Tarski, fast standard inference methods can be applied. One of the major problems—depending on the environment, the robots' capabilities and the tasks to be solved—is the choice of the granularity of an applied calculus. To present, granularity had to be chosen at the start and could not be changed on the fly. In a dynamically changing environment under real time conditions it is necessary to choose a coarse but still adequate granularity of the spatial representation: only in that case irrelevant feature changes fail to trigger unnecessary inference steps. A qualitative, coarse abstraction suppresses tiny changes in the environment and leads to fast computation.

1 Introduction

Most robots currently used for research issues are equipped with a broad variety of fairly reliable sensors. Edutainment robots however often have only low quality sensors. Despite this, they have become increasingly popular and must be able to solve complex spatial tasks even when accurate distance and orientation information is not obtainable. Qualitative reasoning may allow them to do so.

Qualitative Reasoning about space abstracts from the physical world and enables computers to make predictions about spatial relations, even when a precise quantitative information is not available [Cohn, 1997]. The two main trends in Qualitative Spatial Reasoning are topological reasoning about regions [Cohn, 1997; Renz and Nebel, 1998] and positional reasoning about point configurations [Freksa, 1992; Schlieder, 1995]. Positional reasoning, i.e. distance and orientation, in particular is important for robot navigation [Musto *et al.*, 1999].

Calculi dealing with such information have been well investigated over recent years and provide sound reasoning strategies, e.g. about topological relations between regions as in RCC-8 [Randell and Cohn, 1989; Randell *et al.*, 1992], about the relative position orientation of three points as in Freksa's Double Cross Calculus [Freksa, 1992] or about orientation of two line segments as in the Dipole Calculus [Moratz *et al.*, 2000; Schlieder, 1995]. Standard constraint-based reasoning techniques can be applied for reasoning with calculi such as the above mentioned ones. For example, Schlieder [1995] sketched how a qualitative calculus like the Dipole Calculus might be applied to robot navigation.

One of the major problems is the choice of the granularity of an applied calculus according to the environment, the robots' capabilities and the tasks that have to be solved. To present, this granularity had to be chosen in the beginning and could not be changed on the fly. In a dynamically changing environment under real time conditions it is necessary to choose a coarse yet adequate granularity of the spatial representation: only in that case will irrelevant feature changes fail to trigger unnecessary inference steps. A qualitative, coarse abstraction suppresses tiny changes in the environment and results in fast computation.

With the Oriented Point Relation Algebra $\mathcal{OPRA}_m$ we present a calculus whose granularity is scalable with a parameter $m \in \mathbb{N}$. The parameter can be adjusted according to perception and motion capabilities. The reasonable maximum, i.e. the finest reasonable granularity, correlates to the resolution and error of perception and motion. Yet, it would be unwise to use the finest resolution possible just to answer a question whether an object is to the left or right. We present an integration schema where data represented in different granularities can be mixed when deriving new relations from prior observations. The rest of the paper is organized as follows: After a brief introduction of related qualitative spatial calculi and their according properties, we will introduce the $\mathcal{OPRA}_m$ calculus. First we will give a definition for the coarsest type ($m = 1$), followed by the model for arbitrary $m \in \mathbb{N}$ including the rules for composition of base relations. In the end we will give an example with linguistic commands and coarse perceived configuration information that have to be integrated by constraint propagation to achieve survey knowledge.

2 Related Work

Qualitative Spatial Reasoning (QSR) is an abstraction that summarizes similar quantitative states into one qualitative characterization. From a cognitive perspective the qualitative method *compares* features of the domain rather than *measuring* them in terms of some artificial external scale [Freksa, 1992]. The two main directions in QSR are topological reasoning about regions, e.g. the RCC-8 [Randell and Cohn, 1989; Randell *et al.*, 1992], and positional (distance and orientation) reasoning about point configurations. An overview is given in [Cohn and Hazarika, 2001]. We will concentrate on the most important positional calculi for our work.

The Double Cross calculus [Zimmermann and Freksa, 1996] is an approach based on fundamental knowledge about human spatial reasoning. In contrast to previous approaches the base relations do not only describe a relative point position wrt. a single point, but wrt. a vector. In other words, an observer tries to relate to a point C while he is walking from position A to B. In [Scivos and Nebel, 2001] it was shown that the calculus is not closed under permutation and composition, and that reasoning with a set of base relations is NP-hard. A further application driven development based on the scheme above is the Ternary Point Configuration Calculus (TPCC) [Moratz *et al.*, 2003]. We will describe this calculus in more detail in section 4.1.

Schlieder [1995] proposed a calculus with 14 basic relations based on line segments with clockwise or counter clockwise orientation of generating starting points. Isli and Cohn [1998] presented a ternary algebra for reasoning about orientation containing a tractable subset of base relations. Schlieder's approach was extended for robot navigation tasks in [Moratz *et al.*, 2000; Dylla and Moratz, 2005], resulting in relation algebras in the sense of Tarski [Ladkin and Maddux, 1994] at different levels of granularity.

Clementini *et al.* [1997] introduced a binary approach for dealing with qualitative relations at an arbitrary level of granularity. The angles are not necessarily equidistant. Their approach did not provide a general and restrictive schema for reasoning with multiple position expressions. Also no concept for combining relations at different levels was given.

In [Renz and Mitra, 2004] the Star Calculus, a qualitative direction calculus with arbitrary granularity, was introduced. The relation of two points in the plane with respect to one global reference direction is expressed, which leads to $4m+1$ basic relations. These basic relations form a relation algebra for the cases with uniform angles. The authors claim that when using a Star Calculus with more than two reference lines, the boundary between qualitative and quantitative representation disappears. The main disadvantage of the Star Calculus is its need for a global reference direction which must always be available at each point in space.

The extended panorama approach was presented in [Wagner *et al.*, 2003]. The representation is based on the cyclic ordering information of a 360° view within the reference frame of an observing agent and on qualitative distance information. It can be interpreted as an ordered set of relations between an oriented object and the according observed point. Due to this structure it is rotational and translational invariant. Updating the model due to changes in a dynamic environment can simply be done by changing the order. Different levels of granularity were also introduced. No formal method for granularity switches or composition of local observations into survey knowledge was given.

3 The Oriented Point Relation Algebra ($\mathcal{OPRA}_m$)

Objects and locations are represented as simple, featureless points in aforementioned approaches on orientations. In contrast, our paper presents a positional calculus which uses more complex basic entities: It is based on objects which are represented as oriented points. It is closely related to a previously designed calculus which is based on straight line segments (dipoles) [Moratz *et al.*, 2000]. In [Dylla and Moratz, 2005] the dipole approach was extended for modeling behavior in dynamic environments. Conceptually, our new calculus can be viewed as a transition from oriented line segments with concrete length to line segments with infinitely small length. In this conceptualization the length of the objects no longer has any importance. Thus, only the direction of the objects is modeled. *O-points*, our term for oriented points, are specified as pair of a point and a direction on the 2D-plane.

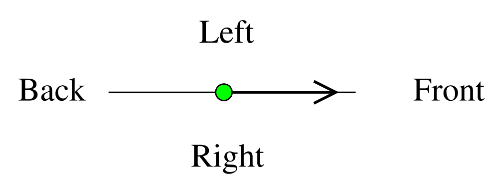


Figure 1: An oriented point and its qualitative spatial relative directions

3.1 Reasoning with Coarse O-Point Relations

In the coarsest representation a single o-point induces the sectors depicted in figure 1. "Front" and "Back" are linear sectors. "Left" and "Right" are half-planes. The position of the point itself is denoted as "Same". A qualitative spatial relative orientation relation between two o-points is represented by the sector in which the second o-point lies with respect to the first one and by the sector in which the first one lies with respect to the second one.

For the general case of the two points having different positions we use the concatenated string of both sector names as the relation symbol. Then the configuration shown in figure 2 is expressed with the relation $A\ \mathrm{RightLeft}\ B$. If both points share the same position the relation symbol starts with the word "Same" and the second substring denotes the direction of the second o-point with respect to the first one as shown in figure 3.

Altogether we obtain 20 different atomic relations (four times four general relations plus four with the o-points at the same position). These relations are jointly exhaustive and pairwise disjoint (JEPD). The relation SameFront is the identity relation. We use $\mathcal{OP}_1$ to refer to the set of 20 atomic relations, and $\mathcal{OPRA}_1$ to refer to the power set of $\mathcal{OP}_1$ which contains all 2^{20} possible unions of the atomic relations.

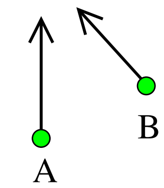


Figure 2: Qualitative spatial relation between two oriented points at different positions. The qualitative spatial relation depicted here is A RightLeft B (which reads: B is to the right of A, and A is to the left of B).

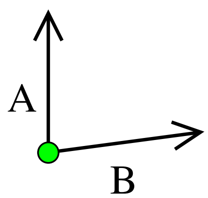


Figure 3: Qualitative spatial relation between two oriented points located at the same position. The qualitative spatial relation depicted here is A SameRight B (which reads: A and B are at the same location, and B is heading right with respect to A).

For reasoning about the o-point relations we apply constraint-based reasoning techniques which were originally introduced for temporal reasoning [Allen, 1983] and also proved valuable for spatial reasoning [Renz and Nebel, 1998; Isli and Cohn, 2000]. In order to apply these techniques to a set of relations, the relations must form a relation algebra [Ladkin and Maddux, 1994], i.e. the atomic relations must be jointly exhaustive and pairwise disjoint and they must be closed under composition ($\circ$), intersection ($\cap$), complement ($\bar{\cdot}$), and converse ($\smile$). There must also be an empty relation, a universal relation, and an identity relation. While the converse, the complement, and the intersection of relations can be computed from the set-theoretic definitions of the relations, the composition of relations must be computed based on the semantics of the relations. The compositions are usually computed only for the atomic relations and then stored in a composition table. The composition of compound relations can be obtained as the union of the compositions of the corresponding atomic relations. The compositions of the atomic relations can be deduced directly from the geometric semantics of the relations (see section 3.4).

O-point constraints are written as xRy where x, y are variables for o-points and R is a $\mathcal{OPRA}_1$ relation. Given a set Θ of o-point constraints, an important reasoning problem is deciding whether Θ is *consistent*, i.e., whether there is an assignment of all variables of Θ with dipoles such that all constraints are satisfied (a *solution*). We call this problem OPSAT. OPSAT is a Constraint Satisfaction Problem (CSP) [Mackworth, 1977] and can be solved using the standard methods developed for CSPs with infinite domains (see, e.g., [Ladkin and Maddux, 1994]).

A partial method for determining inconsistency of a set of constraints Θ is the *path-consistency method*, which enforces path-consistency on Θ [Mackworth, 1977]. A set of constraints is path-consistent if and only if for any two consistent variable instantiations, there exists an instantiation of any third variable such that the three values taken together are consistent. It is necessary but not sufficient for the consistency of a set of constraints that path-consistency can be enforced. A naive way to enforce path-consistency is to strengthen relations by successively applying the following operation until a fixed point is reached:

$$\forall i, j, k: \quad R_{ij} \leftarrow R_{ij} \cap (R_{ik} \circ R_{kj})$$

where i, j, k are nodes and R_{ij} is the relation between i and j. The resulting set of constraints is equivalent to the original set, i.e. it has the same set of solutions. If the empty relation occurs while performing this operation, Θ is inconsistent, otherwise the resulting set is path-consistent.

3.2 Finer Grained O-Point Calculi

The design principle for $\mathcal{OPRA}_1$ can be generalized to calculi $\mathcal{OPRA}_m$ with arbitrary $m \in \mathbb{N}$. Then an angular resolution of $\frac{2\pi}{2m}$ is used for the representation (a similar scheme for absolute direction instead of relative direction was recently designed by Renz and Mitra [2004]).

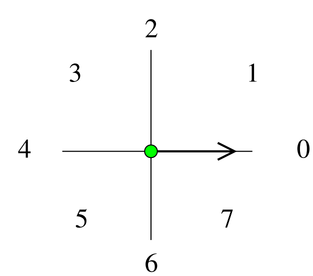


Figure 4: $\mathcal{OPRA}_2$ granularity

To formally specify the o-point relations we use two-dimensional continuous space, in particular $\mathbb{R}^2$. Every o-point S on the plane is an ordered pair of a point $\mathbf{p}_S$ represented by its Cartesian coordinates x and y, with $x, y \in \mathbb{R}$ and and a direction ϕ_S.

$$S = (\mathbf{p}_S, \phi_S), \qquad \mathbf{p}_S = ((\mathbf{p}_S)_x, (\mathbf{p}_S)_y)$$

We distinguish the relative locations and orientations of the two o-points A and B expressed by a calculus $\mathcal{OPRA}_m$ according to the following scheme. We use the symbol φ_{AB} for

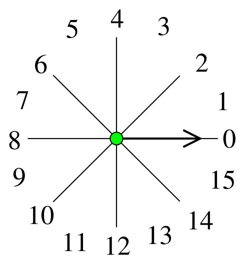


Figure 5: $\mathcal{OPRA}_4$ granularity

$\tan^{-1}\frac{(\mathbf{p}_B)_y-(\mathbf{p}_A)_y}{(\mathbf{p}_B)_x-(\mathbf{p}_A)_x}$ ($\tan^{-1}$ has two arguments, the numerator, and the denominator, and maps to the interval $[0, 2\pi]$). Figures 4 and 5 show the resulting granularity for $m = 2$ and $m = 4$. According to the cyclic order of the directions it is appropriate to enumerate them by using the $4m$ elements of the cyclic group $\mathcal{Z}_{4m}$.

If $\mathbf{p}_A \neq \mathbf{p}_B$ the relation $A\ {}_m\angle_i^j\ B$ $(i, j \in \mathcal{Z}_{4m})$ reads like this: Given a granularity m, the relative position of B with respect to A is described by j and the relative position of A with respect to B is described by i.

Formally, it represents the following set of configurations:

$$
\begin{array}{rl}
 & \left(\left((i \equiv_2 1) \wedge \left(2\pi\frac{i-1}{4m} < \varphi_{AB} - \phi_A < 2\pi\frac{i+1}{4m}\right)\right)\right. \\
\vee & \left.\left((i \equiv_2 0) \wedge \left(\varphi_{AB} - \phi_A = 2\pi\frac{i}{4m}\right)\right)\right) \\
\wedge & \left(\left((j \equiv_2 1) \wedge \left(2\pi\frac{j-1}{4m} < \varphi_{AB} - \phi_B < 2\pi\frac{j+1}{4m}\right)\right)\right. \\
\vee & \left.\left((j \equiv_2 0) \wedge \left(\varphi_{AB} - \phi_B = 2\pi\frac{j}{4m}\right)\right)\right)
\end{array} \tag{1}
$$

$a \equiv_2 b$ stands for $a \mod 2 = b \mod 2$. Using this notation, a simple manipulation of the parameters yields the converse operation $({}_m\angle_j^i)^{\smile} = {}_m\angle_i^j$.

If $\mathbf{p}_A = \mathbf{p}_B$, the relation $A\ {}_m\angle i\ B$ represents the following set of configurations:

$$
\begin{array}{rl}
 & \left((i \equiv_2 0) \wedge \left(\phi_B - \phi_A = 2\pi\frac{i}{4m}\right)\right) \\
\vee & \left((i \equiv_2 1) \wedge \left(2\pi\frac{i-1}{4m} < \phi_B - \phi_A < 2\pi\frac{i+1}{4m}\right)\right)
\end{array} \tag{2}
$$

Hence the relation for two identical o-points $A = B$ for arbitrary $m \in \mathbb{N}$ is $A_m\angle 0B$. Using this notation a simple manipulation of the parameters yields the converse operation $({}_m\angle i)^{\smile} = {}_m\angle(4m - i)$. The composition tables for the atomic relations of the $\mathcal{OPRA}_m$ calculi can be generated using a schema which is based on the parameters m, i, j of the corresponding relations (analogous to the generating scheme for the converse operation). We describe the schema for the composition operation in section 3.4.

To clarify the notation above we will give examples here. The configuration in figure 1 with $m = 1$ for example results in $A\ {}_1\angle_3^1\ B$. Front in this schema is denominated with 0, Left is 1, Back is 2 and Right is 3. In figure 6 the same configuration is shown with the reference frame for $m = 2$. This results in relation $A\ {}_2\angle_7^1\ B$. Thus we can say that B lies in segment 7 regarding A and A lies in segment 1 relative to B. For $m = 4$ (figure 6) we get $A\ {}_4\angle_{13}^3\ B$.

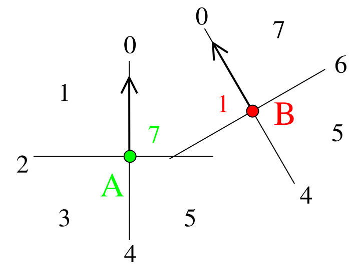


Figure 6: Two o-points in relation $A\ {}_2\angle_7^1\ B$

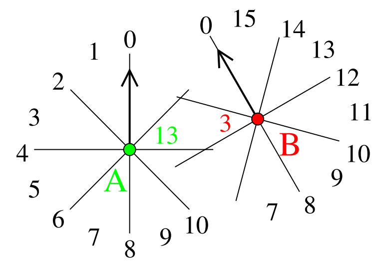


Figure 7: Two o-points in relation $A\ {}_4\angle_{13}^3\ B$

3.3 The Triangle Constraint

Besides the composition we have an additional source for spatial knowledge. The following scene is given: An agent is at position A and perceives object C including the view angle relative to the current heading. Then the agent turns towards position B, moves there and perceives the relative angle to object C again. We now are able interpret this setting as a triangle (compare figure 8). α is defined by the difference of the original heading, the view angle and the heading towards B. β is determined accordingly after the perception. Due to general knowledge about triangles ($\alpha + \beta + \gamma = \pi$) we are able to derive γ.

With A_B we denote the o-point positioned at A and orientated towards position B. In the following, i, k and the according arithmetic operators are still defined in $\mathcal{Z}_{4m}$. Within $\mathcal{OPRA}_m$, α now may be described as

$$A_C\ {}_m\angle k\ A_B$$

and respectively β as

$$B_A \ {}_m\angle i \ B_C \quad .$$

Assuming A, B, C being ordered in mathematically positive orientation and $k \equiv_2 0 \vee i \equiv_2 0$, we can conclude angle γ:

$$C_B \ {}_m\angle(2m - k - i) \ C_A \quad .$$

Thus we can generate additional relations for C with two prior perceptions.

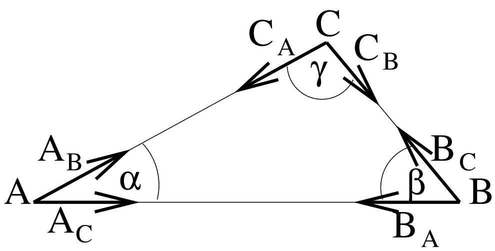


Figure 8: A triangle defined by the o-points A,B and C

3.4 Composition of Relations

Throughout this section we assume that three o-points A, B, C and the relations $A {}_m\angle_i^j B$ and $B {}_m\angle_k^l C$ are given. First we also assume that $\mathbf{p}_A \neq \mathbf{p}_B \neq \mathbf{p}_C$.

In the case of uneven i, j, k and l they correspond to open angular intervals according to (1). ${}_m\angle_i^j$ is called a *totally planar* relation, if $i \equiv_2 1 \wedge j \equiv_2 1$. If $(i + j) \equiv_2 1$, ${}_m\angle_i^j$ is called a *semi-planar* relation. ${}_m\angle_i^j$ is called a *linear* relation, if $i \equiv_2 0 \wedge j \equiv_2 0$.

First we will describe the composition procedure for the special case of totally planar relations, because it is rather straight forward. In the next section we will generalize to a common procedure for arbitrary $\mathcal{OPRA}_m$ relations. In the end we point out how to compose the so-called “same” relations, where two o-points share the same location.

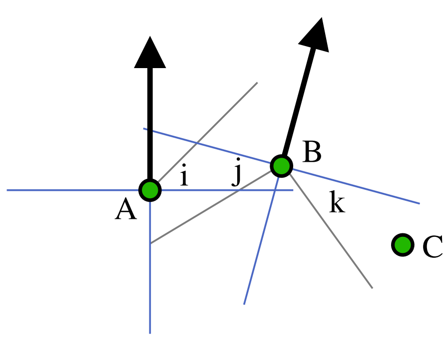


Figure 9: Composition of two $\mathcal{OPRA}_4$ relations $A {}_4\angle_i^j B$ and $B {}_4\angle_k^l C$. In this example the values are $i = 13$, $j = 5$ and $k = 11$ (see figure 5). Because the direction of C is not depicted in this example, no value of l is given. As a result of the composition, C may lie in sectors 9 to 13 with respect to A.

Composition of Totally Planar Relations

Composition of two totally planar relations $A {}_m\angle_i^j B$ and $B {}_m\angle_k^l C$ is mainly a composition of angular intervals. If we want to describe the relative position of C with respect to A, we need to combine the angular intervals which correspond to i, j and k. The first possible sector which can contain C is either i or $i - j + k - 2m - 2$, depending on which one is “first” in a circular order.[1] The last possible sector is either i or $i - j + k - 2m + 2$. To determine this, we define a minimum and a maximum relation within a cyclic group $\mathcal{Z}_n$ ($n \in \mathbb{N}$) with $a, b \in \mathcal{Z}_n$:

$$\min_{\mathcal{Z}}(a,b) = \begin{cases} \min(a,b) & |b-a| < \frac{n}{2} \\ \max(a,b) & |b-a| > \frac{n}{2} \\ b & |b-a| = \frac{n}{2} \end{cases} \tag{3}$$

$$\max_{\mathcal{Z}}(a,b) = \begin{cases} \max(a,b) & |b-a| < \frac{n}{2} \\ \min(a,b) & |b-a| > \frac{n}{2} \\ b & |b-a| = \frac{n}{2} \end{cases} \tag{4}$$

For the sake of simplicity we assume that $\min(a, b)$ is the minimum of the corresponding natural numbers to a and b. $\max_{\mathcal{Z}}(a, b)$ is defined analogously to $\min_{\mathcal{Z}}(a, b)$.

All sectors and linear relations between the first (s_1) and the last possible one (s_2) may contain C. Analogously, we also get a first and a last sector around C which can contain A:

$$\begin{aligned} s_1 &= \min_{\mathcal{Z}}(i, i - j + k - 2m - 2) \\ s_2 &= \max_{\mathcal{Z}}(i, i - j + k - 2m + 2) \\ t_1 &= \min_{\mathcal{Z}}(l, l - k + j - 2m - 2) \\ t_2 &= \max_{\mathcal{Z}}(l, l - k + j - 2m + 2) \end{aligned} \tag{5}$$

We get all possible directions (a full circle) if $s_1 = s_2$ or $t_1 = t_2$, because a composition of totally planar relations can never result in a single sector:

$$s_1' = \begin{cases} s_1 & s_1 \neq s_2 \\ 0 & s_1 = s_2 \end{cases} \qquad s_2' = \begin{cases} s_1 & s_1 \neq s_2 \\ 4m-1 & s_1 = s_2 \end{cases}$$

t_1' and t_2' are derived analogously.

We achieve a disjunction of relations in which C can be with respect to A and a disjunction of relations in which A can be with respect to C. The overall result is a disjunction of all possible combinations:

$$A \ {}_m\angle_i^j \ B \ \circ B \ {}_m\angle_k^l \ C = \bigvee_{a=s_1'}^{s_2'} \bigvee_{b=t_1'}^{t_2'} A \ {}_m\angle_a^b \ C \tag{6}$$

[1] This notation, of course, is simplified: We need to consider m an element of the cyclic group as well, but we did not want to introduce another symbol for this purpose.

Composition of Arbitrary Relations

In this section we present a generalized schema for determining the composition of arbitrary $\mathcal{OPRA}_m$ relations. The only cases to be excluded are the "same" relations, which are described in the following section, and those resulting in a linear sector or a disjunction of two linear sectors:

$$((j = k + 2m) \vee (j = k)) \wedge j \equiv_2 0 \wedge k \equiv_2 0 \qquad (7)$$

The solution for these few cases can be constructed fairly easily. For all other cases the procedure is as follows:

A linear part of an $\mathcal{OPRA}_m$ relation can be seen as an angular interval $[\alpha_1, \alpha_2]$ with $\alpha_1 = \alpha_2$. According to the second and fourth line of (1) the composition formula must be adapted for the cases of even values of i, j, k and l. Therefore a linearity correction term

$$\psi(i, j, k) = \sum_{a \in \{i,j,k\}} ((a + 1) \mod 2) \qquad (8)$$

is incorporated to the equations in (5). ψ counts the number of linear relations. Simply adding (or subtracting) ψ, however, may deliver (half) closed intervals in the case of i or l being even; but this cannot happen. So we can make sure to achieve open intervals by using modified minimum and maximum relations for $\mathcal{Z}_n$ ($n = 4m$ in this case):

$$\min{}'_{\mathcal{Z}}(a, b) = \begin{cases} \min(a, b) & |b - a| < \frac{n}{2}, \min(a, b) \equiv_2 1 \\ \min(a, b) + 1 & |b - a| < \frac{n}{2}, \min(a, b) \equiv_2 0 \\ \max(a, b) & |b - a| > \frac{n}{2}, \max(a, b) \equiv_2 1 \\ \max(a, b) + 1 & |b - a| > \frac{n}{2}, \max(a, b) \equiv_2 0 \\ b & |b - a| = \frac{n}{2}, b \equiv_2 1 \\ b + 1 & |b - a| = \frac{n}{2}, b \equiv_2 0 \end{cases}$$

$$\max{}'_{\mathcal{Z}}(a, b) = \begin{cases} \max(a, b) & |b - a| < \frac{n}{2}, \max(a, b) \equiv_2 1 \\ \max(a, b) - 1 & |b - a| < \frac{n}{2}, \max(a, b) \equiv_2 0 \\ \min(a, b) & |b - a| > \frac{n}{2}, \min(a, b) \equiv_2 1 \\ \min(a, b) - 1 & |b - a| > \frac{n}{2}, \min(a, b) \equiv_2 0 \\ b & |b - a| = \frac{n}{2}, b \equiv_2 1 \\ b + 1 & |b - a| = \frac{n}{2}, b \equiv_2 0 \end{cases}$$

We now get

$$\begin{aligned} s_1 &= \min{}'_{\mathcal{Z}}(i, i - j + k - 2m - 2 + \psi(i, j, k)) \\ s_2 &= \max{}'_{\mathcal{Z}}(i, i - j + k - 2m + 2 - \psi(i, j, k)) \\ t_1 &= \min{}'_{\mathcal{Z}}(l, l - k + j - 2m - 2 + \psi(l, j, k)) \\ t_2 &= \max{}'_{\mathcal{Z}}(l, l - k + j - 2m + 2 - \psi(l, j, k)) . \end{aligned}$$

In contrast to the totally planar cases, a single sector is a possible result when composing semi-planar relations. For discriminating a full circle from a single sector, we need to consider the linearity of the relations given by ψ:

$$s'_1 = \begin{cases} 0 & s_1 = s_2 \wedge \psi(i, j, k) = 0 \\ s_1 & \text{else} \end{cases}$$

$$s'_2 = \begin{cases} 4m - 1 & s_1 = s_2 \wedge \psi(i, j, k) = 0 \\ s_1 & \text{else} \end{cases}$$

and analogously for t'_1 and t'_2.

The resulting $\mathcal{OPRA}_m$ relation is

$$A \ {}_m\angle_i^j \ B \circ B \ {}_m\angle_k^l \ C = \bigvee_{a=s'_1}^{s'_2} \bigvee_{b=t'_1}^{t'_2} A \ {}_m\angle_a^b \ C \quad . \qquad (9)$$

Composition with "Same" Relations

Compositions of cases where either $\mathbf{p}_A = \mathbf{p}_B$ or $\mathbf{p}_B = \mathbf{p}_C$, is rather simple, because it can be seen as an addition of intervals, or, if $i \equiv_2 0 \wedge k \equiv_2 0$, vectors.

$${}_m\angle i \circ {}_m\angle_k^l = \begin{cases} \bigvee_{a=s_1}^{s_2} {}_m\angle_a^l & i \equiv_2 0 \wedge k \equiv_2 0 \\ i + k & \text{else} \end{cases} \quad , \qquad (10)$$

$$\begin{aligned} s_1 &= i + k - 1 + \psi(i, k, 1) \\ s_2 &= i + k + 1 - \psi(i, k, 1) \end{aligned}$$

ψ again denotes the linearity term given in (8). The third argument is 1 because we only need two arguments here.

The composition ${}_m\angle_i^j \circ {}_m\angle k$ works analogously. Composition of two "same" relations is trivial.

3.5 Integration of Relations with Different Granularity

Sometimes it is reasonable to perceive or act using different degrees of accuracy depending on context or time constraints. Therefore we have relations at different levels of granularity, i.e. varying m. It is not reasonable to represent such information at a very precise level, because a large disjunction with many literals would emerge. We call the chosen m a *context dependent granularity*. Inconsistencies arising due to imprecise or faulty perception or movement can be solved by adding even more uncertainty to draw a reasonable conclusion.

Given two relations with granularity m_1 and m_2, it is no problem to integrate relations with $m_1 = n * m_2$ with $n \in \mathbb{N} > 0$ and $m_1 > m_2$. If the values are not a multiple of each other, naive and fast methods for integrating the knowledge are e.g. the least common multiple (LCM) or the greatest common divisor (GCD). Information loss is minimal with the LCM, but again a large disjunction might be generated. In contrast, combining the relations with the GCD of m_1 and m_2 results in a greater loss of information, but the result consists of fewer relations compared with the LCM approach. Currently, we choose a method where the relations are combined according to their algebraic semantics and a suitable granularity is chosen depending on the result.

4 Qualitative Spatial Reasoning in Robotics

We will now give a detailed example on how to integrate local knowledge into survey knowledge with the presented TPCC calculus. Afterwards we will show how the given problem can be solved with $\mathcal{OPRA}_m$ as well. The example we use here has already been introduced in [Dylla and Moratz, 2004].

The basis of the example is a robot system able to perceive colored cubes. The system only measures the direction towards perceived objects. It cannot measure their distance.

Furthermore the system is able to perform discrete motion steps. The task is to "move to the red object behind the blue cube". The initial situation is shown in figure 11(a). For better differentiation we visualize the two ambiguous red objects $B1$ and $B2$ as circles. First we will give a short recap of TPCC [Moratz *et al.*, 2003]. Then we show how to solve the given task with TPCC, followed by a solution with $\mathcal{OPRA}_m$.

4.1 The Ternary Point Configuration Calculus (TPCC)

TPCC [Moratz *et al.*, 2003] deals with point-like objects in the 2D-plane. It is an application oriented variant of the Double Cross calculus [Freksa, 1992], which allows for finer distinctions of positional information than most calculi for constraint based reasoning presented before. The partition of the calculus is shown in figure 10.

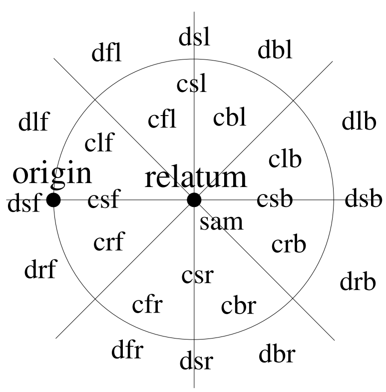


Figure 10: The reference system used by TPCC

The letters f, b, l, r, s, d, c stand for front, back, left, right, straight, distant, and close, respectively. The terms front, back, etc. are given for mnemonic purposes only. The use of TPCC relations in natural language applications is shown in an article by Moratz *et al.* [2002]. In this application TPCC relations are used for natural human robot interaction. The configuration in which the referent is at the same position as the relatum is called *sam* (for "same location"). The two special configurations in which origin and relatum have the same location (*dou*, *tri*) are also base relations of this calculus.

For a formal and precise definition of the relations the corresponding geometric configurations on the basis of a Cartesian coordinate system represented by $\mathbb{R}^2$ were described. For example, the special cases for the three points $A = (x_A, y_A)$, $B = (x_B, y_B)$ and $C = (x_C, y_C)$ are defined as follows:

$$\begin{aligned} A, B \text{ dou } C \quad &:= \quad x_A = x_B \wedge y_A = y_B \wedge \\ & \qquad (x_C \neq x_A \vee y_C \neq y_A) \\ A, B \text{ tri } C \quad &:= \quad x_A = x_B = x_C \wedge y_A = y_B = y_C \end{aligned}$$

For the cases with $A \neq B$ a relative radius $r_{A,B,C}$ and a relative angle $\phi_{A,B,C}$ must be defined[2]:

$$\begin{aligned} r_{A,B,C} \quad &:= \quad \frac{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}} \\ \phi_{A,B,C} \quad &:= \quad \tan^{-1} \frac{y_C - y_B}{x_C - x_B} - \tan^{-1} \frac{y_B - y_A}{x_B - x_A} \end{aligned}$$

Then we have spatial relations as the examples shown below. All relations are named in figure 10 except the special cases *dou* and *tri*. For a complete list of the definitions we refer to [Moratz *et al.*, 2003].

$$\begin{aligned} A, B \text{ sam } C \quad &:= \quad r_{A,B,C} = 0 \\ A, B \text{ clb } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge 0 < \phi_{A,B,C} \leq \frac{1}{4}\pi \\ A, B \text{ dlb } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge 0 < \phi_{A,B,C} \leq \frac{1}{4}\pi \\ A, B \text{ cfl } C \quad &:= \quad 0 < r_{A,B,C} < 1 \wedge \frac{1}{2}\pi < \phi_{A,B,C} < \frac{3}{4}\pi \\ A, B \text{ dsr } C \quad &:= \quad 1 \leq r_{A,B,C} \wedge \phi_{A,B,C} = \frac{3}{2}\pi \end{aligned}$$

TPCC is not closed under transformations (intersection, complement and converse), i.e. a transformation might generate a proper subset of base relations. It is as well not closed under strong composition ($\circ$):

$$\forall A, B, D : A, B(r_1 \circ r_2) D \leftrightarrow \exists C : A, B(r1) C \wedge B, C(r_2) D$$

Therefore 4-consistency cannot be enforced directly when inferring with TPCC. Instead a weak composition ($\circledast$) was defined:

$$\forall A, B, D : A, B(r_1 \circledast r_2) D \leftarrow \exists C : A, B(r1) C \wedge B, C(r_2) D$$

The composition table for the weak case was already presented in [Moratz *et al.*, 2003]. The weak operations are still sufficient to solve a task as shown in our example in the next subsection.

4.2 A Solution with TPCC

With the relations defined in TPCC the task "move to the red cube behind the blue cube" can be described such that one of the relations clb, csb or crb must hold for $(C, R1, B1)$ or $(C, R1, B2)$. We will refer to the disjunction of the three relations as $c?b$. We visualize the initial situation in figure 11(a). Figure 11(b) integrates the initially perceived constraints about what is known about $B1$ and $B2$. To deduce the desired knowledge the agent has to move. How to choose the most reasonable action for a maximum of information gain goes beyond the scope of this paper. Therefore we apply the heuristic: "Move straight forward until the first object is passed and get new perceptions there".

We will use a simple path-based scheme of constraint propagation, where the two last relations of a path are composed

[2] Here we refer to the arcus tangent function with two arguments mapping all four quadrants (atan2).

and then the reference system is incrementally moved towards the beginning of the path to demonstrate reasoning efficiently.

In the example the robot moves towards a position to the right of the blue cube (fig. 11(c)–11(d)). In figure 11(e) it reaches the desired position ($R2$). Relation 3 just describes the fact that the agent's move to the right of the blue cube relative to the starting point $R1$. The agent's perception gives additional knowledge on $B1$ and $B2$ relative to $(C, R2)$[3]. In order to make a composition we have to transform relation 3 with the **Sc** transformation leading to relation 3' (fig. 11(f)). Now 3' can be composed with 5 leading to the fact that $c?b$ is not valid for $(R1, C, B2)$ (fig. 11(g)). Composing 3' and 4 shows $B1$ being somewhere behind the blue cube relative to $(R1, C)$ (fig. 11(h)). Although according to constraint propagation $B1$ might be somewhere left of the reference axis, $B1$ is the only red object having a chance of fulfilling the given constraint ($c?b$).

Solving general constraint satisfaction networks on the basis of Double Cross relations is NP-hard [Scivos and Nebel, 2001]. TPCC has not yet been proven to be NP-complete. Anyway, in the case of many real world problems the desired knowledge can be gained in polynomial time without the need to solve the whole constraint system. The solution can be obtained via a path-based constraint propagation as presented in [Dylla and Moratz, 2004]. All the algorithms given there are in P.

4.3 $\mathcal{OPRA}_m$— Reasoning about Perceptions

At first the agent perceives basic relations between the objects of the environment. Then the agent moves, gets new perceptions, and can combine these perception using qualitative spatial reasoning using the previously defined operations of $\mathcal{OPRA}_m$. We now relate to the example in figure 11. According to the granularity of TPCC we assume $m = 4$. A_B denotes the o-point at position A with orientation towards B. In contrast, $_BA$ denotes the inverting, i.e. point A looking away from object B.

The initial task (figure 11(b)) may be expressed as

$$R1_C \ {}_4\angle_0^0 \ C_{R1} \wedge C_{R1} \ {}_4\angle_{\{7-9\}}^{\{0-15\}} \ BX_* \ ?$$

with $X \in \{1, 2\}$ and with $A \ {}_m\angle_{\{i-j\}}^{\{k-l\}} \ B$ denoting the disjunction

$$\bigvee_{a=i}^{j} \bigvee_{b=k}^{l} A \ {}_m\angle_a^b \ B \quad .$$

The $*$ stands for the set of all available points in our setting. We do this, because the orientation of BX is of no interest for the given task.

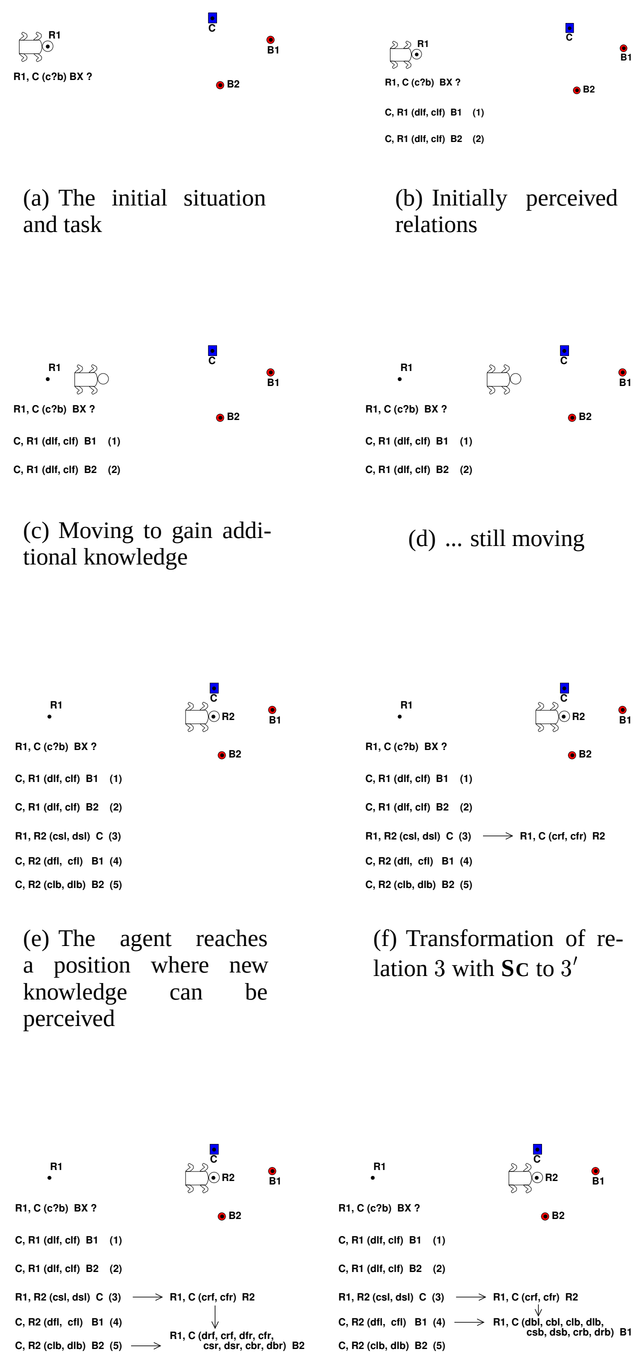
The initial perceptions (figure 11(b)) are:

(1) $R1_{R2} \ {}_4\angle 1 \ R1_C \rightarrow R1_{R2} \ {}_4\angle_1^0 \ C_{R1}$

(2) $R1_{R2} \ {}_4\angle 1 \ R1_{B1} \rightarrow R1_{R2} \ {}_4\angle_1^0 \ B1_{R1}$

(3) $R1_{R2} \ {}_4\angle 15 \ R1_{B2} \rightarrow R1_{R2} \ {}_4\angle_{15}^0 \ B2_{R1}$

[3] Perhaps more relations are perceivable, but we concentrate on the relations relevant for this example.



(a) The initial situation and task

(b) Initially perceived relations

(c) Moving to gain additional knowledge

(d) ... still moving

(e) The agent reaches a position where new knowledge can be perceived

(f) Transformation of relation 3 with **Sc** to 3′

(g) Path-based integration of 3′ with 5, resulting $B1$ being to the right of C

(h) Integration of 3′ with 4 resulting in $B2$ being somewhere behind C

Figure 11: Solving the task: "Go to the red object (circle) behind the blue cube!" with TPCC

So far $R2$ is just an abstract point in the direction of the robot's current orientation. For additional knowledge the agent moves to the real position $R2$, which is the rectangular point of intersection of a straight move and the first object passed according to our heuristics.

(4) $R1_{R2}\ {}_4\angle^8_0\ {}_{R1}R2 \rightarrow R1_{R2}\ {}_4\angle^0_0\ R2_{R1}$

At $R2$ the Aibo perceives (figure 11(e))

(5) ${}_{R1}R2\ {}_4\angle 4\ R2_C \rightarrow {}_{R1}R2\ {}_4\angle^0_4\ C_{R2}$

(6) ${}_{R1}R2\ {}_4\angle 1\ R2_{B1} \rightarrow {}_{R1}R2\ {}_4\angle^0_1\ B1_{R2}$

(7) ${}_{R1}R2\ {}_4\angle 13\ R2_{B2} \rightarrow {}_{R1}R2\ {}_4\angle^0_{13}\ B2_{R2}$

From (5) follows

(5') $R2_{R1}\ {}_4\angle 12\ R2_C \rightarrow R2_C\ {}_4\angle 4\ R2_{R1}$

Applying the triangle constraint to (1) and (5') we now are able to derive

(8) $C_{R1}\ {}_4\angle 3\ C_{R2}$

Again we must transform (5) to

(5") $C_{R2}\ {}_4\angle^4_0\ {}_{R1}R2$

Now we can compose (5") and the "same" relation (8) resulting in

(9) $C_{R1}\ {}_4\angle^4_3\ {}_{R1}R2$

Deriving the relative position of $B1$ needs the composition of semi-planar relations (9) and (6), and with (7) for $B2$ respectively.

(10) $C_{R1}\ {}_4\angle^{\{11-15\}}_{\{3-9\}}\ B1_{R2}$

(11) $C_{R1}\ {}_4\angle^{15}_{\{3-5\}}\ B2_{R2}$

In (11), compared to our initial task, one can see that $B2$ is definitely being positioned somewhere to the left of $C1$ regarding the orientation towards our starting position $R1$. Although $B1$ might also be somewhere to the left regarding the same reference system, it is the only red object having the chance to fulfill the initial constraint.

5 Conclusion

We presented a calculus for representing and reasoning about qualitative relative orientation information. Oriented points serve as the basic entities since they are the simplest spatial entities that have an intrinsic orientation. We identified systems of atomic relations on different granularity levels between o-points and identified a scheme for computing the calculi's operation tables based on their geometric semantics. It turns out that our calculus is a relation algebra in the sense of Tarski. Therefore fast standard constraint-based reasoning methods can be applied under real time conditions. The granularity of the calculus allows to suppress irrelevant feature changes in dynamically changing environments.

Potential applications of the calculus are demonstrated by a robotics scenario. In the scenario, linguistic commands and coarse perceived configuration information have to be integrated by constraint propagation to get survey knowledge. The accuracy of the calculus permits robotics applications, in particular in cognitively driven scenarios featuring linguistic communication and approximate visual perception.

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