The Spatial and the Visual in Mental Spatial Reasoning: An Ill-Posed Distinction

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Abstract. It is an ongoing and controversial debate in cognitive science which aspects of knowledge humans process visually and which ones they process spatially. Similarly, artificial intelligence (AI) and cognitive science research, in building computational cognitive systems, tended to use strictly spatial or strictly visual representations. The resulting systems, however, were suboptimal both with respect to computational efficiency and cognitive plausibility. In this paper, we propose that the problems in both research strands stem from a misconception of the visual and the spatial in mental spatial knowledge processing. Instead of viewing the visual and the spatial as two clearly separable categories, they should be conceptualized as the extremes of a continuous dimension of representation. Regarding psychology, a continuous dimension avoids the need to exclusively assign processes and representations to either one of the categories and, thus, facilitates a more unambiguous rating of processes and representations. Regarding AI and cognitive science, the concept of a continuous spatial / visual dimension provides the possibility of representation structures which can vary continuously along the spatial / visual dimension. As a first step in exploiting these potential advantages of the proposed conception we (a) introduce criteria allowing for a non-dichotomic judgment of processes and representations and (b) present an approach towards representation structures that can flexibly vary along the spatial / visual dimension.

Introduction

In this contribution we address issues of modeling visual and spatial aspects in mental reasoning with spatial information. More specifically, we focus on information on how a certain number of objects can be related to one another in space, with regard to topology, distance, and directional relations.

Whether the corresponding mental representations of such information can be more adequately described by means of non-modal structures such as propositions (Jahn, 2003) or one-, two- and three-dimensional arrays (e.g., Ragni, Knauff, & Nebel, 2005; Glasgow & Papadias, 1992), or through diagrams (Chandrasekaran, Kurup, Banerjee, Josephson, & Winkler, 2004) or images (Bertel, Barkowsky, Engel, & Freksa, 2006; Kosslyn, 1994) is the topic of much ongoing research. While the group of non-modal structures is frequently linked to reasoning with spatial mental models (Johnson-Laird, 1983), diagrams and images are usually employed to capture mental representations evoked in mental imagery (Kosslyn, 1994). In terms of working memory processes, spatial mental models are often described as mainly relying on central executive, non-modal functions (Gillhooly, Logie, Wetherick, & Wynn, 1993), though not exclusively (De Vooght & Vandierendonck, 1998), while mental imagery has been suggested to heavily tap visuo-spatial subsystems (Kosslyn & Thompson, 2003; Ishai & Sagi, 1997; Kosslyn & Sussman, 1995).

Consequently, for spatial reasoning, tasks are usually modeled as *either* employing spatial or visual mental representations (i.e., being based on spatial mental models or on images), but rarely both. This is somewhat in-line with double dissociations found to exist between visual and spatial short-term memory (Klauer & Zhao, 2004). Also, existing comparative approaches of image- and model-based reasoning with same or comparable tasks are aimed at dissociating the two conditions (e.g. Knauff & Johnson-Laird, 2002, for relational reasoning) rather than establishing if, how, and when spatial and visual processing co-occurs.

On a neuropsychological level, evidence has long been collected for a functional and anatomical separation of two major pathways in higher-level visual processing (Ungerleider & Mishkin, 1982; Haxby et al., 1991). Originating occipitally, a ventral pathway runs to the inferior temporal lobe and processes object properties such as color and shape (thus labeled “what” pathway), while a dorsal pathway projects to posterior parietal areas and processes spatial attributes and movements (“where” pathway).

Inter-individual differences have been reported for the use of spatial and visual representations, for example, for abstract problems (Hegarty & Kozhevnikov, 1999) or during wayfinding (Aginsky, Harris, Rensink, & Beusmans, 1997). Such differences led to the postulation of different individual cognitive styles, such as “verbalizers”, “spatializers”, or “visualizers”. More recently, object and spatial subgroups have been proposed for the visualizers (Kozhevnikov, Kosslyn, & Shephard, 2005; Kozhevnikov, Hegarty, & Mayer, 2002), leading to further differentiations.

Given all this evidence for *different types* of mental representations, it hardly comes as a surprise that even for spatial reasoning tasks mental representations have been often modelled as either visual or spatial and that, frequently, properties of either format have been investigated independently of the other format. Consequently, a main research focus has been on dissociating between rather than on integrating different representation formats in spatial reasoning. However, while the dissociation between the visual and the verbal may be adequate for many types of mental reasoning, we argue that it led and still leads to misconceptions about the nature of representations in *spatial* knowledge processing. Instead of viewing the visual and the spatial as two clearly separable categories, for spatial reasoning, they should be conceptualized as the extremes of a continuous dimension of representation with various types of representations in-between.

While it has been shown that for simple relational problems “visual” strategies are often slower than “spatial” ones (Knauff & Johnson-Laird, 2002), visual representations (i.e., mental images) can be suspected to have advantages over spatial mental models in situations where much spatial information needs to be integrated or high degrees of visual detail play role (cf. Kosslyn & Thompson, 2003). Despite strong computational advantages of spatial mental models in terms of parsimony, there exists no elegant computational approach today for flexibly and dynamically integrating topological, distance, and directional knowledge into a spatial (but non-visual) representation format. In the current contribution, we suggest that a continuum of visual / spatial representations may best serve to model the different aspects of mental reasoning with spatial information.

In the following, we first focus on existing conceptions of describing spatial mental knowledge processing and identify two core problems. We then argue for an integrative view of visual / spatial aspects in mental spatial knowledge processing that may overcome the strict separation between the two modalities. We propose scalable representation structures as a modeling conception for describing mental spatial knowledge processing that comprises both spatial and visual aspects. Finally, we briefly discuss a number of open issues to be addressed in future research.

Existing Conceptions and Problems

Mirroring the above stated distinction between the visual and the spatial, research in psychology as well as in AI and cognitive science have mainly treated the spatial and the visual as clearly separable aspects of tasks. In doing so, psychological research has mainly concentrated on the question of which (properties of) entities are–or even must be–represented or processed either visually or spatially (cf. Kosslyn & Thompson, 2003) and the question of which individual traits of a person might indicate her capability to represent or process entities visually or spatially (cf. Kozhevnikov, Hegarty, & Mayer, 2002).

A major focus of AI and cognitive science on the other hand has been on the conceptualization of different types of representation structures for building artificial cognitive systems (cf. Glasgow & Papadias, 1992) or for modeling natural cognitive systems like humans (cf. Barkowsky, 2002). In particular, the representation structures proposed so far were usually aimed at being either visual representations or spatial representations.

Thus, much research in psychology and AI / cognitive science has–at least implicitly–assumed a strict distinction between visual and spatial aspects in mental spatial knowledge processing. On closer inspection, however, there are at least two problems associated with such a stance, namely (1) that there currently exists no consensus as to what the defining characteristics of visual or spatial mental representations really are, and (2) that current artificial cognitive systems and models of mental spatial knowledge processing either employ strictly spatial or strictly visual representations. These problems will be briefly outlined in the following two sections.

Problem 1 – Spatial and Visual: Where’s the Difference?

A crucial prerequisite for categorizing entities with respect to their visual or spatial nature are clear criteria for judging entities as being visual or spatial. Similarly, to distinguish people regarding their capability to represent or process entities visually or spatially requires tests which selectively tap this capability. To construct such tests, again criteria are needed to decide which test items are instrumental in determining the relevant ability. Such criteria, however, currently do not seem to exist.

On the one hand there is lack of clarity regarding what prevalent tests like the mental rotation test (Vandenberg & Kuse, 1978) and the Minnesota Paper Form Board (Likert & Quasha, 1941) actually measure. The former, for instance, was intended to measure the ability to visualize and mentally manipulate spatial arrangements such as three-dimensional block figures. In accord with this conception, the test has subsequently been employed in several studies (e.g., Charlot, Tzourio, Zilbovicius, Mazoyer, & Denis, 1992; Mellet, Tzourio, Denis, & Mazoyer, 1995) to identify the imagery (i.e., visualization) abilities of study participants. Yet, other researchers (e.g., Kosslyn & Thompson, 2003) have claimed that the mental rotation test does only measure *spatial* reasoning abilities, but not visualization capabilities. Remarkably, none of these works mentions clear criteria on which grounds the test is thought to measure visual or spatial abilities. Rather, both the researchers utilizing the test to measure visualization and the researchers utilizing the test to measure spatial abilities seem to assume it being obvious that the test measures what they utilize it for.

In addition, there is a more general disaccord with respect to the question of which (properties of) entities are spatial and which ones are visual. Levine, Warach, and Farah (1985), for example, assume that an analog clock face is something visual, whereas Kosslyn and Thompson (2003) judge clock faces and the position of the corresponding clock hands as being spatial. A similar confusion exists with respect to the shape of objects. In some work (cf. Kosslyn & Thompson, 2003), shape is assumed to be a prime example for a visual property. Other researchers have, however, argued that shape can be and is represented and processed spatially (e.g. Leeuwenberg, 2004). Like with the employment of tests, criteria-based justifications for one or the other stance are hard to find in the literature.

These examples illustrate that unambiguous, objective criteria for judging entities as being visual or spatial currently do not seem to exist. Consequently, categorizing entities and persons according to the visual / spatial dichotomy has given rise to a number of different categorizations and to some confusion regarding the characteristics of the visual and the spatial. Such conceptual disarray poses a serious problem for investigating and realizing the processes and representations in spatial knowledge processing in natural and artificial cognitive systems.

Problem 2 – Integrating Spatial Representations

Numerous ways of representing and processing spatial knowledge have been devised in AI both for explanatory purposes in cognitive models and for technical purposes in diagrammatic reasoning systems. The most widely used 2-dimensional structure is the regular rectangular raster array (e.g., Glasgow & Papadias, 1992; Khenkhar, 1991; Ragni, Knauff, & Nebel, 2005). This type of representation structure supports, for instance, translation operations, neighborhood-based processes like region growing operations, or topological relations. To more adequately model operations like rotation, scaling, or symmetry detection, Funt (1980) used a circular representation structure consisting of individual processors arranged in the form of concentric rings. Other types of representation which have been proposed are, for example, vector images (Barkowsky, 2002), bitmap images (Kosslyn, 1980) or less complex qualitative or metrical linear ordering structures (e.g., Schlieder & Berendt, 1998; Chang, Shi, & Yan, 1987).

Despite this diversity, two aspects are characteristic for nearly all of the approaches in AI and cognitive science. First, the representation structures devised so far often are applicable only to specific types of spatial knowledge. Put differently, such representation structures are only appropriate to represent certain types of spatial knowledge, for example just knowledge about orientation or just knowledge about topology. As a consequence, most of the existing structures do not allow representing spatial situations which require integrating several types of spatial knowledge. Notable exceptions from this rule are representation structures like bitmap or vector images which have been termed diagrammatic representations: by depicting the spatial situation in a 2D plane all aspects essential to spatial knowledge processing (i.e., distance, orientation, topology, shape) can be represented in an integrated way. Consider, for example, the following situation: Larissa returns from a holiday and tells her friend that she has just been to a beautiful place called San Giovanni a Piro. As Larissa's friend has never been to San Giovanni a Piro, she asks Larissa about the location of the place. Larissa's description could sound as follows: San Giovanni a Piro is a small village in the south of Italy on the Tyrrhenian coast, not far from Naples but not as far as the tip of the boot of Italy.

There are several kinds of spatial knowledge involved here and they have to be considered in combination to determine the location of San Giovanni a Piro (`SGP`). The topological relations are: `in (Italy, SGP)` and `touches (SGP, Tyrrhenian coast)`. A cardinal direction relation is `south (SGP, central Italy)`, distance information includes `not far from (SGP, Naples)` and `not as far from as (SGP, Naples, tip of boot of Italy)`. Last, there is shape information involved, such as `shape of (tip of boot of Italy)`.

Second, like in psychological research, AI and cognitive science work does seem to underlie–at least implicitly–the assumption that each representation structure is necessarily either spatial or visual. Therefore, computational systems are either restricted to the processing of specific types of spatial knowledge (where integration of different knowledge types is not required), or they comprise several representation structures some of which are spatial and some of which are visual. A common approach in the latter type of systems is to utilize the (specific) spatial representation structures wherever this is sufficient and to employ the visual representations for situations with greater complexity or to benefit from their higher specificity (e.g., Chandrasekaran et al., 2004).

However, utilizing distinct spatial and visual representations of the same situation concurrently entails to either store all information about a spatial situation redundantly or to newly and fully construct a visual representation from the knowledge already stored in the spatial representations whenever needed. In both cases the system would be suboptimal with respect to computational efficiency. Moreover, it seems implausible that such a computational system is a good model of human cognition, because this would be in disaccord with the idea that information processing in the human mind is organized according to optimize the information processing demands (cf. Collins & Quillian, 1969).

Regarding cognitve plausibility one might argue that both entertaining multiple representations and redundantly storing information are fundamental properties of cognition in some contexts and, accordingly, the procedure employed in current computational systems is not necessarily cognitively implausible. Such argument, however, neglects the fact that evidence for multiple representations in spatial cognition usually suggests that these representations differ considerably with respect to their content (e.g., Brockmole & Wang, 2002; Carlson, 1999). In other words, if multiple representations store information redundantly they do so only partially. On the contrary, the usual computational approaches would suggest that the human cognitive system would maintain multiple representations such that some representations do not hold any information not already contained in some other representation. Such an assumption seems to depart too far from cognitive efficiency too be accepted without explicit empirical support.

Thus, in such “hybrid” approaches the problem of computational inefficiency and–in the case of cognitive science–also of cognitive implausibility arises.

Lessons learned

The arguments in the preceding two sections illustrate two crucial problems associated with the spatial vs. visual distinction: first, there is currently no consensus as to what the defining characteristics of visual or spatial mental representations really are. As a consequence, psychological research reports ambiguous results with respect to (a) which entities are processed visually and which ones are processed spatially, and (b) which individual traits indicate an inclination to process information visually or spatially. Second, current artificial cognitive systems and models of mental spatial knowledge processing either employ strictly spatial or strictly visual representations. This seems, on the one hand, computationally inefficient and, on the other hand, cognitively implausible.

In the following, we propose an approach to tackle both of these problems. The basis for our approach is the observation that both problems can be traced back to the tendency existing in psychology as well as AI and cognitive science of trying to strictly separate the spatial from the visual. We claim that such a strict distinction is (a) not possible and (b) not reasonable. In our view the spatial and the visual are not two separable categories, but should better be viewed as two extremes of a continuous dimension. Taking such a position allows addressing the above mentioned issues.

First of all, assuming a continuous visual / spatial dimension explains why it was not possible until now to achieve a clear and unequivocal categorization of (properties of) entities and a person’s processing mode as being either spatial or visual: some entities, for instance the mental representations of the clock faces and clock hands discussed above–which have been claimed to comprise visual as well as spatial aspects–, could be assumed to be lying somewhere well in-between the two extremes of the visual / spatial dimension and therefore cannot be exclusively assigned to either of the two categories. Accordingly, a more appropriate characterization of entities on the visual / spatial dimension would be a comparative one which allows identifying whether an entity is more spatial or more visual than one or several others.

Second, regarding AI systems and cognitive models, the concept of a continuous dimension suggests to employ not only representation structures which are either spatial or visual; rather, representation structures should be employed which lie in-between the two extremes. Such "intermediate" structures integrate spatial and visual aspects and potentially allow making a smoother transition between strictly spatial and strictly visual representation structures. To illustrate this consider the following example: A computational system starts working on a spatial reasoning task for which it employs a spatial representation structure R_{S1} (e.g., containing topological knowledge). Assume further that at some point during reasoning the integration of another representation structure R_{S2} (e.g., containing directional knowledge) becomes necessary to represent the situation. One option would be to now create a strictly visual representation R_V (i.e., a picture) and to copy all information contained in R_{S1} and R_{S2} to R_V. Format changes would be required for all content copied (e.g., from a specific spatial format to a specific visual format). A different option that utilizes the postulated visual / spatial continuum of representations would involve slight changes to R_{S1} and R_{S2} and the association of their respective content; it could lead to the formation of an integrated spatial representation R_I that also permits to represent the new situation, however, at lower computational costs than R_V would require because only little recoding and copying would be needed in its creation. In our view, integrated spatial representations potentially provide for lean (i.e., economic) and cognitively plausible representation structures.

Summing up, the problems associated with the spatial / visual distinction in psychology, AI, and cognitive science can be avoided by assuming a continuum underlying the spatial / visual dimension. However, such an assumed continuum raises several new questions like, for example, as to what the criteria for determining the position of an entity on the continuum really are or how one can realize intermediate representation structures in a computational system. In the next section, we will give first answers to these and related questions thus evolving a new conception of the spatial / visual distinction as well as introducing first ideas towards a new type of representation structures for spatial reasoning in computational systems.

Beyond a Strict Separation of Spatial and Visual

As explained in the previous section a strict separation of spatial and visual which implies two categories with clear cut boundaries entails a number of serious problems. One way to avoid these problems is to assume a continuous dimension with spatial and visual as extremes. Such bipolar conception is, however, only useful if there are clear criteria based on which one can–at least in an ordinal way–place (properties of) entities and / or representation structures unambiguously on this dimension. Such criteria will be detailed in the following.

Criteria for the Spatial / Visual Dimension

On the basis of the properties of cases where visual representation structures have been employed in computational systems or of situations in which humans have been assumed to use visual mental systems we propose four criteria for the placement of spatial representations on the spatial / visual dimension. In other words, in most or all circumstances (a) which necessitated the use of visual representation structures in computational systems or (b) in which psychological studies detected the employment of visual processing, the following four criteria were more strongly pronounced than when no visual processing took place:

- a high number of types of spatial relations included in the representation,
- a high number of different spatial relations included in the representation,
- a high degree of specificity of the relationship (i.e. the degree of completeness of the set of relations) of each pair of included entities, and
- a high degree of exemplarity of entities and relations included in the representations.

Each of the criteria is proportionally related to the visual side of the continuum, that is, for example, the higher the specificity the closer is the corresponding entity or representation supposed to be to the visual endpoint of the spatial / visual dimension.

In the rest of this section the four criteria will be detailed in turn.

Number of Types of Relations This criterion refers to the types of spatial relations involved in the mental processing of a spatial reasoning tasks. As argued above the spatial representation structures usually employed in computational systems are knowledge type specific, that is, are intended to and only permit to represent one kind of spatial knowledge like, for instance, orientation knowledge. As soon as several types of knowledge have to be or are represented in an integrated way, normally diagrammatic representation structures are used. Accordingly, we assume that processes and / or representation structures are the more visual the more types of relations are involved. Essentially, this means that more integrated and thus more complex processes and representations are viewed as being more visual.

Number of Relations Due to their integrated nature, visual processes and visual representations tend to comprise not only more types of relations, but also deal with a higher number of relations than spatial processing and spatial representations, respectively. That is, in an integrated, visual representation typically more spatial relations are–if only implicitly–specified between the represented entities. Therefore, our second criterion to judge an entity's / a representation's position along the spatial visual dimension is the number of relations involved.

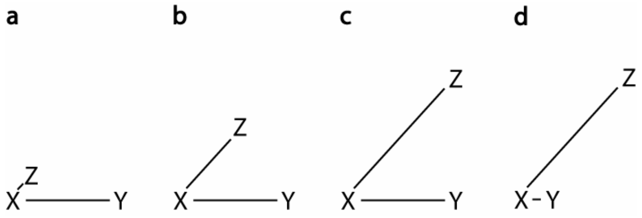

Fig. 1. Four possible interpretations of the situation *X* is west of *Y* and *Z* is northeast of *X*.

Specificity Knowledge about spatial situations may–and often does–allow for several interpretations. Consider, for example, the following (spatial) facts a person might know about the three entities *X*, *Y*, and *Z*: *X* is west of *Y* and *Z* is northeast of *X*. Although these two facts may be quite helpful for the task the person tries to accomplish the precise position of the three entities to each other is not unambiguously given. Knowing just the two above stated facts, the orientation relation between *Y* and *Z* could be *Z* is west of *Y*, *Z* is northwest of *Y*, *Z* is north of *Y*, or *Z* is northeast of *Y* (see Fig. 1a, b, c, and d, respectively). In other words the two given facts do not fully specify the spatial situation between the three entities with respect to their mutual orientation. However, depicting the two facts in the two-dimensional plane as in Fig. 1 ineluctably specifies the relation between *Z* and *Y*. Thus, one defining characteristic of visual processes and representations is their higher specificity when compared to more ambiguous representations (cf. Stenning & Oberlander, 1995). Consequently, we chose specificity as a further criterion for locating processes and representations on the spatial / visual dimension.

Exemplarity In psychology, certain aspects of entities like color, shape, or texture have been assumed to entail more visual processing (e.g., Knauff & Johnson-Laird, 2002; Levine, Warach, & Farah, 1985). Likewise, visual representation structures used in spatial knowledge processing in AI and cognitive science are usually more appropriate for representing such details as specific shapes and relations than spatial representation structures are (cf. Barkowsky, 2002).

At the same time, such details will be so much more likely to be processed or represented visually the more the reasoning process involves concrete exemplars instead of prototypes / categories of objects. If, for example, one needs to reason about the internal layout of a familiar office building (say, for navigating in it) it is more probable that the reasoning process will involve color, shape, and texture information about that building than if one reasons only about the internal structure of office buildings in general.

As a result of these two observations, we propose that the extent to which concrete exemplars (instead of prototypes / categories) are part of the reasoning process forms one criterion for judging processes and representations with respect to their location on the spatial / visual dimension (cf. categorical and coordinate pathways, Kosslyn, 1994).

To illustrate the criteria and how they might be applied, consider the two representations given in Fig. 2a and 2b, respectively, of the same spatial situation comprising a church, a pond, and a tree. The representation in Fig. 2a just encodes two spatial relations between the three objects, namely that the tree is left of the church and that the pond is in front of the church. In contrast, the representation in Fig. 2b represents a number of additional spatial relations between the three objects, for example the information that the pond is in front of the tree, the distance information that both the tree and the pond are close to the church, and the additional directional information that the church is right of the tree and behind the pond. Accordingly, the second representation does not only encode more spatial relations between the objects, but also more types of spatial relations. Furthermore, whereas the first representation does not specify the precise position of the three objects (apart from the fact that the tree is left of and the pond is in front of the church), the second one does. Finally, the second representation can be seen as encoding concrete exemplars of the categories tree, church and pond, since the church, for instance, is rather small and warped (which can be assumed not to be the properties of a prototypical church). Thus, compared to the representation in Fig. 2a the representation in Fig. 2b has more types of spatial relations, more spatial relations, a higher specificity, and a higher exemplarity and, therefore, will be judged to be more visual than the first representation.

This example shows that the criteria proposed both nicely allow locating representations on the visual-spatial dimension and lead to judgments which are in accord with previous categorizations in which the representation of Fig. 2b would have been more visual than the representation in Fig. 2a. Consequently, the first problem arising in assuming a continuous spatial / visual dimension, namely devising consistent criteria which allow continuously judging processing / representations, has been satisfactorily engaged. Yet, the second problem, that is, what kind of representation structures might occupy intermediate position on the spatial / visual dimension has not been discussed until now. In the following section, aspects pertaining to this second problem will be presented.

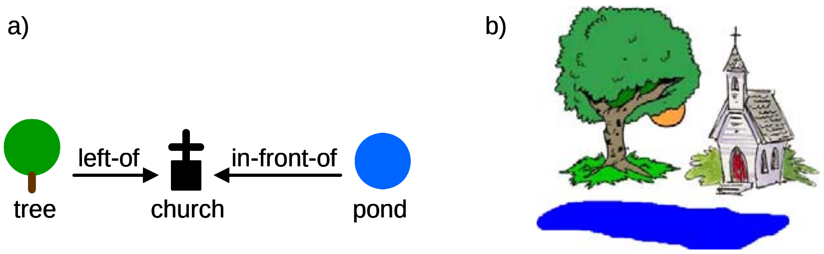

Fig. 2. Two representations of spatial relations between a church, a pond, and a tree.

Scalable Representation Structures

As argued above, assuming a spatial / visual continuum does not only avoid category assignment problems, but also potentially permits to realize computationally more efficient and cognitively more plausible computational systems. A prerequisite to the latter, however, is the availability of representation structures which are located between the extreme end points of the dimension, that is, representation structures which are neither exclusively spatial nor exclusively visual. Ideally, these representation structures should be such that they allow incrementally building a more and more complex / visual representation of spatial situations, based on the current task demands. Due to their incremental, demand driven refinability we term such representation structures *scalable representation structures*.

Although existing representation structures for computational cognitive systems may vary in the amount of information they represent (i.e., in their complexity), they usually do so confined to one type of spatial knowledge, that is, are not scalable across knowledge types. In fact, scalable representation structures in the sense just mentioned currently do not exist. Due to the advantages scalable representation structures might yield, this seems to be a serious lack of current research and, as a consequence, we took first steps in devising such structures.

Before further describing our approach, however, one fundamental issue needs to be addressed: are scalable representation structures to be conceived of (a) as one single representation structure incrementally being upgraded and growing more and more in complexity or (b) as a system of separate distributed specific representation structures which are more and more tightly combined on demand? In the next section these two possibilities will be discussed.

Starting from the results of this discussion we will present a more detailed account of the requirements for scalable representation structures as well as first ideas towards their realization in the subsequent section before finally discussing open issues with respect to our approach and their potential merit for psychological research.

One for All or All for One? The defining characteristics of scalable representation structures are that they (a) are incrementally refinable on demand and (b) can occupy intermediate locations on the spatial / visual dimension. From these characteristics alone, however, it is not clear how scalable representation structures should be conceived of regarding their organization. On the one hand, scalable representation structures could be viewed as being *monolithic*, that is, being constituted by a single representation structure which is–at least potentially–able to integrate all (types of) spatial relations at once. Regarding the aim of building a computational model of human mental spatial knowledge processing such a stance would, furthermore, posit that human cognition relies on such a monolithic representation structure. On the other hand, scalable representation structures could also be conceived of as being *composed*, that is, consisting of several distinct smaller (and more specific) representation structures. Since the representation structures to be devised are meant to improve existing and / or to facilitate the building of new computational cognitive systems, we chose to base our decision regarding the two possibilities of conceptualizing scalable representation structures on already existing evidence from psychological as well as AI /cognitive science research.

Regarding psychology, it seems to be a common assumption that spatial knowledge about the environment is stored in the form of several "…smaller chunks, each of which is encoded by a separate representation." (Brockmole & Wang, 2002). And indeed there are several studies indicating that humans maintain different representations for spatial reasoning tasks with respect to (a) the content of these representations (i.e., different hierarchically representations; see, for instance, Hirtle & Jonides, 1985; McNamara, 1986), (b) whether object information (what) or location information (where) is represented (e.g., Hegarty & Kozhevnikov, 1999; Klauer & Zhao, 2004), and (c) whether relations are represented egocentrically or allocentrically (Easton & Sholl, 1995; Rieser, 1989). Thus, there seems to be ample psychological evidence supporting composed scalable representation structures.

This evidence is further corroborated by the AI / cognitive science literature. Numerous different types of representation structures both for diagrammatic reasoning systems (e.g., Chandrasekaran, Kurup, Banerjee, Josephson, & Winkler, 2004) and cognitive models (e.g., Ragni et al., 2005) have been proposed so far. Notably, it turned out that the proposed structures individually only support a certain (small) number of representational requirements. Put differently, until now AI / cognitive science researchers have not been able to devise a single, monolithic representation structure for spatial reasoning. In contrast, it has been claimed (cf. Sloman, 1985) that several special purpose representation structures are more sensible than one general purpose representation.

In accord with these converging evidence from the two fields of research we assume that scalable representation structures should be conceptualized a being composed of several specialized representation structures. A more detailed account of how scalable representation structures may be constructed from a number of distinct, dedicated representation structures will be given in the next section.

Integration Approaches As indicated by the above section there seems to be ample evidence and common agreement that spatial information is represented distributed across several separate representation structures. There is, however, considerable less work regarding the question how these separate representation structures could be combined or integrated such that spatial reasoning across the full range of possible spatial relations could be realized, since previous research efforts have mainly concentrated on specific aspects of spatial reasoning in isolation from each other solely considering, for example, cardinal directions (e.g., Frank, 1995) or topological relations (Egenhofer, 1994). One notable exception to this rule is the work of Sharma (1996), who develops a formalism to combine topological and direction relation for spatial inferences in Geographic Information Systems (GIS). Similarly, the TPCC calculus (Moratz, Nebel, & Freksa, 2003) combines directional and distance knowledge in a formal framework.

Although these are important approaches regarding the integration of several knowledge types in spatial reasoning, there are at least three shortcomings associated with these approaches with respect to scalable representation structures: first, though, these approaches combine some of the separate types of spatial knowledge they normally do not encompass all types (e.g., Sharma, 1996, neglects distance information). Second, the approaches are meant to be solutions to some technical problem. Consequently, their main focus is on technical aspects and, in particular, they do not try to be and are not cognitively plausible. Third, the reasoning formalisms proposed do not include a specification of the structures in which the combined knowledge types are represented, that is, the approaches do not assume / specify any particular representation structures. To build a computational model of mental spatial knowledge processing, however, (a) representation structures need to be devised, (b) all types of knowledge have to be integrated, and (c) both former points have to be realized in a cognitively plausible way. Despite several cognitive models of spatial reasoning such a cognitive account of scalable representation structures currently does not seem to exist. As with the more technical approaches, previous cognitive models of spatial reasoning seem to have concentrated merely on single types of knowledge (see, e.g., Ragni et al., 2005; Goodwin & Johnson-Laird, 2005).

Accordingly, a computational account of integrated mental spatial knowledge processing currently does not exist. Since this seems to be a serious lack, we are currently working on developing such an account. As the first step in that development, we chose to concentrate on scalable representation structure, that is, how separate knowledge types might be represented and integrated in a cognitively adequate way.

Like explicated in the previous section, it seems most plausible to conceive scalable representation structures as arising from the interplay of distributed, separate representation structures which are combined on demand. Yet, such a conception entails the problem of coordinating the individual structures, that is, the individual knowledge represented in these structures. Coordination is necessary, because the different types of knowledge are dependent on one another. If, for instance, some entity *X* is known to be a proper part of some other entity *Y*, and *Y* is known to be north of a third entity *Z*, the direction relation between *X* and *Z* can not be arbitrary. Thus, if (the combination of) the separate representation structures should form a coherent representation of some spatial situation the consistency of the knowledge stored in the individual representation structure needs to be ensured. Any conception of scalable representation structures therefore needs to include some mechanism(s) which realize(s) this mutual adjustment. Before one can devise such mechanisms the mutual dependencies of the different knowledge types must be known. Accordingly, our first step towards developing scalable representation structures was to identify these dependencies by building composition tables across different knowledge types.

Table 1. Composition table giving the possible topological relations between *Y* and *Z* (shown in the corresponding table cells) given both a topological relation between *Y* and *X* ($Y\ R_1\ X$) and a distance relation between *X* and *Z* ($X\ R_2\ Z$); $\mathcal{TOP}$ is the set of all possible topological relations; see text for details.

$X\ R_2\ Z$ $Y\ R_1\ X$	**cl**	**md**	**fr**
equal	$\mathcal{TOP}$	disjoint	disjoint
disjoint	$\mathcal{TOP}$	$\mathcal{TOP}$	$\mathcal{TOP}$
tangent	$\mathcal{TOP}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$
overlaps	$\mathcal{TOP}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$
in/t	$\mathcal{TOP}$	disjoint	disjoint
in/nt	$\mathcal{TOP}$	disjoint	disjoint
contains/t	$\mathcal{TOP}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$
contains/nt	$\mathcal{TOP}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$	$\mathcal{TOP}\setminus\{$equal, in t/nt$\}$

Composition tables in general state all relations between two entities *Y* and *Z* which are possible in the light of given relations between *X* and *Y* as well as *X* and *Z*. As discussed above and shown in Fig. 1, for example, given *X* south-west of *Z* and *X* west of *Y* possible direction relations[1] between *Y* and *Z* are *Z* west of *Y*, *Z* north-west of *Y*, *Z* north of *Y*, and *Z* north-east of *Y*. A complete composition table not only states the possible relations for one pair of relations that hold between Y and X and X and Z, but for all potential pairs of relations. Furthermore, like in this example, composition tables usually are concerned with only one type of knowledge, that is, the given relations as well as the determined possible (given one pair) relations belong to

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[1] Using cardinal directions and assuming an 8-sector model with the directions north, north-west, west, south-west, south, south-east, east, and north-east.

the same type of spatial knowledge (direction in the presented example). Yet, to identify the mutual dependencies between different types of knowledge such composition tables are not sufficient. Instead, one needs to take into account which relations of a certain knowledge type K_a are possible in the light of two relations R_1, R_2 where R_1 is of type K_a and R_2 is of another knowledge type. Table 1 presents an example of such a composition table across knowledge types. Shown in the table cells are those topological relations which could hold between *Y* and *Z* given the topological relation indicated by the relation in the corresponding row of the first column and the distance relation given in the corresponding column of the first row. For example, if *Y* and *X* are equal and *X* and *Z* have medium (third column) or far (fourth column) distance to each other *Y* and *Z* have to be disjoint.

Analogously to the construction of Table 1, we constructed cross composition tables for all six possible pairings of types of spatial knowledge, that is, distance / direction, direction / distance, distance / topology, topology / distance, topology / direction, and direction / topology. For each of these pairings we distinguished which of the two paired knowledge types was inferred (i.e., which knowledge type was inside the table cells). Thus, overall, 12 composition tables were created. To allow constructing complete, unique, and correct composition tables without the construction process getting out of hand we had to take several additional assumptions. Although these assumptions restrict our approach to a subset of all possible spatial situations, such a procedure seemed justified as a very first step towards scalable representation structures.
The assumptions we took were:

1. Possible distance relations between entities are close (cl), medium (md), and far (fr).
2. Possible direction relations are north (N), east (E), south (S), and west (W).
3. Possible topological relations are equal, disjoint, tangent, overlaps, in/tangential (in/t), in/not tangential (in/nt), contains/tangential (contains/t), and contains/not tangential (contains/nt) which are defined as in the RCC-8 calculus (Cohn et al., 1997).
4. If the distance between two entities is medium or far they have to be disjoint.
5. Two entities can only be close to each other if no other entity lies between them.
6. Entities are two-dimensional convex regions.

Building on these assumptions the 12 cross composition tables allowed identifying the mutual dependencies of the knowledge types which are shown in Table 2. One dependency, for example, is the fact that the direction relation between two entities *X*, *Y* is imposed on the entities contained by *X* / *Y* (see first line in the direction / topology row).

In developing scalable representation structures the crucial issue is to take these dependencies into account, that is, to devise mechanisms that ensure that the rules stated in Table 2 are not violated[2]. But how can such mechanisms be realized, given the assumption that the overall spatial representation is the combination of several distinct partial representations? We assume that the basic cohesion between the distinct sub-

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[2] This is not to say that composition tables or processes utilizing them would be part of mental spatial knowledge processing. The composition tables are only meant to constitute a list of requirements which results the to be developed cognitively plausible mechanisms of integrated mental spatial knowledge processing should tend to yield (see also the Open Issues section below).

structures arises from links connecting the entities which are part of the individual representations. More precisely, those entities in the separate representations that stand for the same object of the represented situation are mutually and completely linked (i.e., every entity in one substructure links to the corresponding entity in all other substructures). Structurally, these links are all that is necessary to realize the functionality of scalable representation structures. First, if during mental spatial knowledge processing the need for considering an additional type of spatial knowledge arises, an appropriate representation (for that knowledge type) can be created and then linked to the already existing representation structures. In this way the overall representation structure can be easily extended on demand and, thus, is truly scalable. Furthermore, according to the rules in Table 2, the links allow realizing the consistency between the individual representation structures. If some new information regarding some entities comes to the knowledge of the cognitive system, that is, this knowledge is incorporated in one of the substructures, corresponding entities in the other substructures can be identified via the links and possible changes to these other structures (due to the dependencies stated above) can be applied. In our current conceptualization of such adjustment procedures we assume sets of representation pair specific processes which realize the adjustment. Put differently, for each pair of knowledge types like, for instance, topology and direction, there exists a set of processes specific to these particular knowledge types which ensure the consistency between the individual representation structures of these types.

Table 2. Mutual dependencies between the different knowledge types as identified by the cross composition tables. *X*, *Y* and *Z* are distinct entities.

Knowledge Types	**Dependencies**
topology / distance	**If** *X* equal *Y* **Then** every *Z* with distance d to *X* / *Y* has distance d to *Y* / *X*
	If *X* has distance *d* to *Y* and *Y* is inside *Z* **Then** *X*'s distance to *Z* cannot be bigger than *d*
	If *X* has distance *d* to *Y* and *Y* contains *Z* **Then** *X*'s distance to *Z* cannot be smaller than *d*
	If *X* has distance md or fr to *Y* and *Y* contains *Z* **Then** *X* and *Z* are disjoint
	If *X* has distance md or fr to *Y* and *Y* is in, overlaps or touches *Z* **Then** *X* cannot contain *Z*
direction / distance	**If** *X* is close to *Y* and *Y* is *dir* of *Z*, where *dir* is a direction relation **Then** *X* is dir_{inv} of *Z* cannot hold, where dir_{inv} is the inverse of *dir*
direction / topology	**If** *X* inside *Y* and *Y* is *dir* of *Z* **Then** *X* is *dir* of *Z* holds
	If *X* equal *Y* and *Y* is *dir* of *Z* **Then** *X* equal *Z* cannot hold.
	If *X* is *dir* of *Y* and *Y* inside *Z* **Then** *X* cannot contain *Z*

Open Issues As stated above, the current conceptions are but a first step towards scalable representation structures, though, in our view, it is a promising one. There are several issues which need further clarification. For example, the mutual dependencies identified so far are based solely on cross composition tables of the form K_a, K_b => K_a / K_b, where K_a and K_b are different knowledge types. Sharma (1996) has termed this type of cross composition *heterogeneous,* and identified two other forms of cross composition which he termed *mixed* (K_b, K_b => K_a) and *integrated* (K_{a1}, K_{b1}; K_{a2}, K_{b2} => K_{a3}, K_{b3}). To give a complete account of dependencies between representations of different knowledge types, it seems necessary to also respect mixed and integrated forms of cross composition.

Apart from these more technical concerns working on the development of scalable representation structures, we identified additional psychological issues: So far, we have described our ideas towards scalable representation structures. To develop a satisfactory cognitive model of spatial reasoning, which is capable of integrating different kinds of spatial knowledge, we need to consider the following questions, which only empirical research can answer: (A) how well are different kinds of spatial knowledge integrated by humans; is the integration symmetric or asymmetric, regarding the ordering of the knowledge fragments to be integrated? (B) Consider, for example, a spatial reasoning task which involves two spatial relations of different types. Are all dependencies between knowledge types described equally respected by humans? Does a resulting inference depend on the order of the given premises? Are some of the dependencies preferred or omitted during mental reasoning? (C) If, for example, one gets to know an orientation relation between two entities, will mental knowledge representation “automatically” entail an inference of distance information (e.g., as it would be the case in diagrammatic reasoning)? How tightly are the different knowledge types coupled? (D) How fast is an integration of additional spatial knowledge with an existing mental representation? What if the knowledge is of a type that, so far, was not included? Considering the limitation of the working memory, how many different representation structures can be successfully integrated? (E) In case of changes in already instantiated knowledge structures, new information has to be updated and communicated among the coupled representation structures. How fast is such propagation of additional knowledge? And again, due to the limitations of memory, do all of the integrated knowledge structures remain consistent, or how many integrated knowledge structures can be kept consistent at a time?

Our cognitive modeling approach aims at specifying scalable representation structures capable of integration of different kinds of spatial knowledge. In doing so, the spatial relations and inference rules have to be made explicit. However, further empirical research is needed to shed light on the visual and the spatial in spatial reasoning and to provide important information on concrete properties of the proposed scalable representation structures.

Conclusions

This contribution has proposed that instead of conceptualizing mental representations in spatial reasoning as either exclusively spatial or visual in format, different representation formats located on a continuum between these two extremes may be more adequate. Four criteria have been identified that are suggested to be positively proportional to visual characteristics of a representation: a high number of types of spatial relations, a high number of different relations, a high degree of specificity of relationships between entities, and a high degree of exemplarity of entities and relations. We have put forward the notion of scalable representation structures whose position on the visual / spatial continuum changes with the integration of additional spatial knowledge. While a first approach that exploits inherent interdependencies between different types of spatial knowledge for mental spatial reasoning is suggested and discussed, a number of open issues remain for further research before adequate scalable spatial representations can be introduced for computational cognitive modeling. It was the aim of our paper to initiate a debate about properties of potential, integrated (i.e., cross-type) representation structures in mental spatial reasoning. Attempts to dissociating between mental faculties often lead to an increase in detailed knowledge; however, trying to understand the mechanisms behind integrating different faculties in reasoning is equally important.

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